

The Bipolar Transistor Model Mextram

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- The Bipolar Transistor Model Mextram
 - Model description
- RF performance
- Scalability and statistical modelling
- The future of Mextram
 - New release in 1999
 - Modelling of SiGe bipolar transistors
- Summary

- Introduction to Mextram
- Equivalent circuit
- Charge modelling
 - Depletion charges, diffusion charges
- Collector current
 - Early effect, high injection
- Base current
 - Forward current gain, reverse current gain
- Avalanche multiplication
- Series resistances
 - Base resistance, collector epilayer resistance
- Miscellaneous

- Mextram has been introduced within Philips in 1985
 - Updates
 - * 1987 Mextram 502
 - * 1993 Mextram 503
 - Physics based
 - Developed for analog applications
 - Suited for both digital and analog applications

- Mextram/MM9 have been introduced in public domain in 1993
 - http://www.semiconductors.philips.com/Philips_Models/

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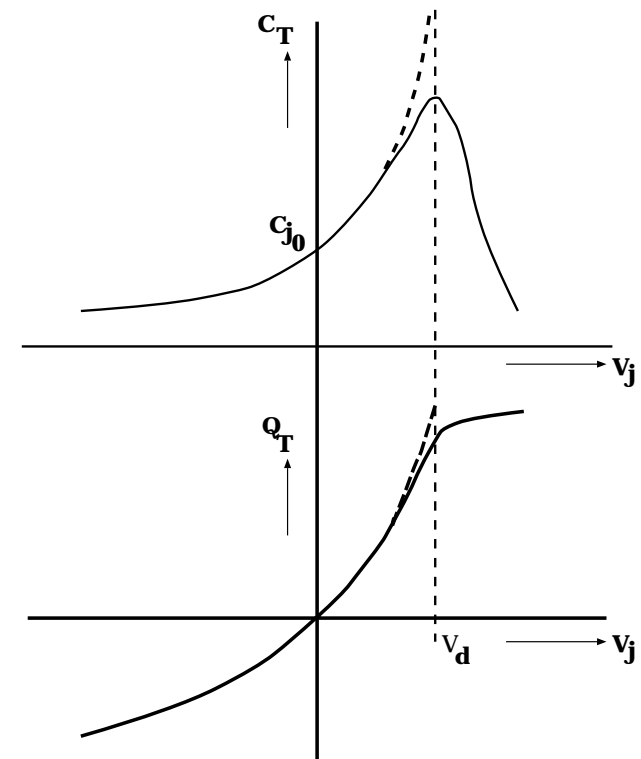
Depletion charges and capacitances

$$C_{DEPL}(V_j) = \frac{C_{j0}}{\left(1 - \frac{V_j}{V_d}\right)^p}$$

$$Q_{DEPL}(V_j) = \int_0^{V_j} C_{DEPL}(V_j) \cdot dV$$

$$= \frac{C_{j0} \cdot V_d}{1 - p} \cdot \left\{ 1 - \left(1 - \frac{V_j}{V_d}\right)^{1-p} \right\}$$

- $Q_{DEPL}(0) = 0$
- singularity at $V_j = V_d$



$$Q_{DEPL}(V_j) = \frac{(1 + K) \cdot C_{j0} \cdot V_d}{1 - p + K} \cdot \left[1 - \frac{(1 + K)^{p/2} \cdot \left(1 - \frac{V_j}{V_d}\right)}{\left\{ \left(1 - \frac{V_j}{V_d}\right)^2 + K \right\}^{p/2}} \right]$$

base-emitter junction:

$$Q_{T_e} = Q_{DEPL}(V_{b2e1}, C_{j_e}, V_{de}, p_e, K = 0.01) \cdot (1 - XC_{j_e})$$

$$Q_{T_e}^s = Q_{DEPL}(V_{b1e1}, C_{j_e}, V_{de}, p_e, K = 0.01) \cdot XC_{j_e}$$

substrate-collector junction:

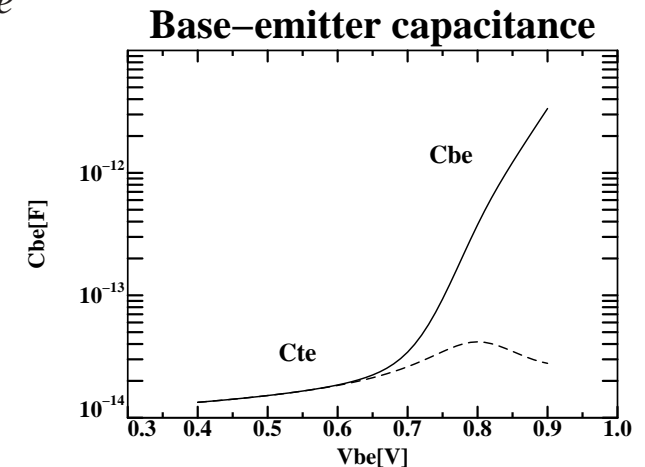
$$Q_{T_s} = Q_{DEPL}(V_{sc1}, C_{j_s}, V_{ds}, p_s, K = 0.01)$$

extrinsic base-collector junction:

takes into account finite thickness of the epilayer: $XP = \frac{x_{d0}}{W_{epi}}$

$$Q_{T_{ex}} = (1 - XC_{j_c}) \cdot [(1 - XP) \cdot Q_{DEPL}(V_{b1c1}, C_{j_c}, V_{dc}, p_c, K = 0.1) + XP \cdot C_{j_c} \cdot V_{b1c1}]$$

base-collector capacitance underneath the emitter depends on collector current !



Diffusion charges: Q_{be} , Q_{bc}

Assumptions:

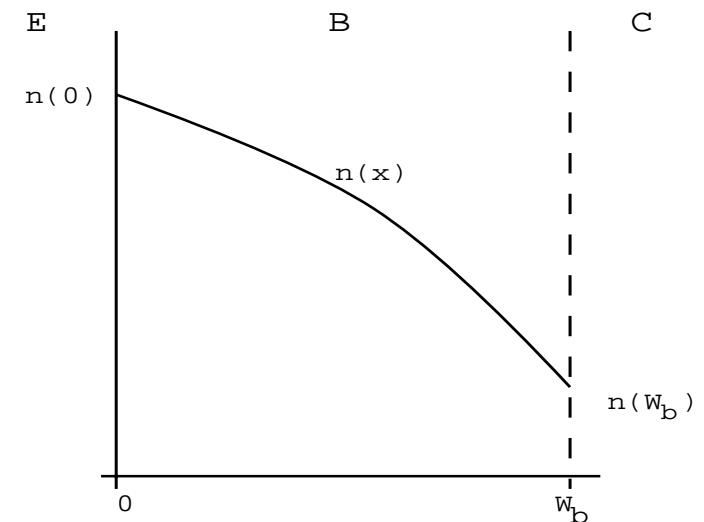
- no recombination in the quasi-neutral base: $J_p = 0$ and $p = n + N_a$
- non-homogeneous base doping: $N(x) = \hat{N}_a \cdot \exp\left(\frac{-\eta \cdot x}{W_b}\right)$

With boundary conditions $n(0)$ and $n(W_b)$ we can derive:

$$n(x) = f(x, \eta) \cdot n(0) + r(x, \eta) \cdot n(W_b)$$

$$Q_{be} = q \cdot A_{em} \cdot n(0) \cdot \int_0^{W_b} f(x, \eta) \cdot dx$$

$$Q_{bc} = q \cdot A_{em} \cdot n(W_b) \cdot \int_0^{W_b} r(x, \eta) \cdot dx$$



with normalized carrier densities: $n_0 = n(0)/\hat{N}_a$, $n_b = n(W_b)/\hat{N}_a$

$$n_0 \cdot (1 + n_0) = \frac{n_i^2}{\hat{N}_a^2} \cdot \exp\left(\frac{V_{b2e1}}{V_t}\right)$$

$$n_b \cdot (e^{-\eta} + n_b) = \frac{n_i^2}{\hat{N}_a^2} \cdot \exp\left(\frac{V_{b2c2}}{V_t}\right)$$

neutral base width ($0 - W_b$) is modulated by the depletion layers:

$$Q_b = Q_{b_0} + Q_{T_e} + Q_{T_c} = Q_{b_0} \cdot (1 + q_1) \quad q_1 \text{ accounts for Early effect}$$

final result:

$$Q_{be} = Q_{b_0} \cdot (1 + q_1) \cdot F(\eta, n_0) \cdot n_0$$

$$Q_{bc} = Q_{b_0} \cdot (1 + q_1) \cdot R(\eta, n_b) \cdot n_b$$

collector epilayer underneath the emitter and extrinsic collector regions:

$$Q_{epi} = I_s \cdot Q_{b0} \cdot \frac{\exp\left(\frac{V_{b2c2}}{V_t}\right) - \exp\left(\frac{V_{b2c1}}{V_t}\right)}{I_{epi}}$$

$$Q_{ex} = \frac{1 - XCj_c}{XCj_c} \cdot (Q_{bc}(V_{b1c1}) + Q_{epi}(V_{b1c1}))$$

emitter charge storage: *parameters:* τ_e, m_τ

$$Q_{ne} = \tau_e \cdot I_s \cdot \left[\exp\left(\frac{V_{b2e1}}{m_\tau \cdot V_t}\right) - 1 \right]$$

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Moll-Ross relation for the main electron current

$$I_n = I_s \cdot \frac{\exp\left(\frac{V_{b2e1}}{V_t}\right) - \exp\left(\frac{V_{b2c2}}{V_t}\right)}{Q_b/Q_{b0}} = \frac{I_f - I_r}{q_b}$$

$$Q_b = q \cdot A_{em} \cdot \int_e^c p \cdot dx = Q_{b0} + Q_{T_e} + Q_{T_c} + Q_{be} + Q_{bc}$$

$$q_b = \frac{Q_b}{Q_{b0}} = 1 + \frac{Q_{T_e} + Q_{T_c}}{Q_{b0}} + \frac{Q_{be} + Q_{bc}}{Q_{b0}}$$

⇓

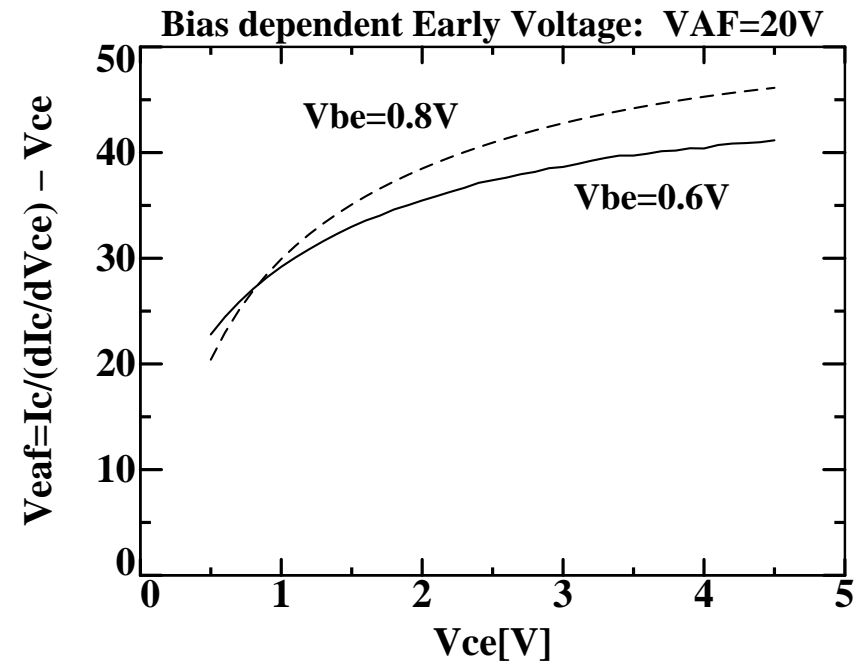
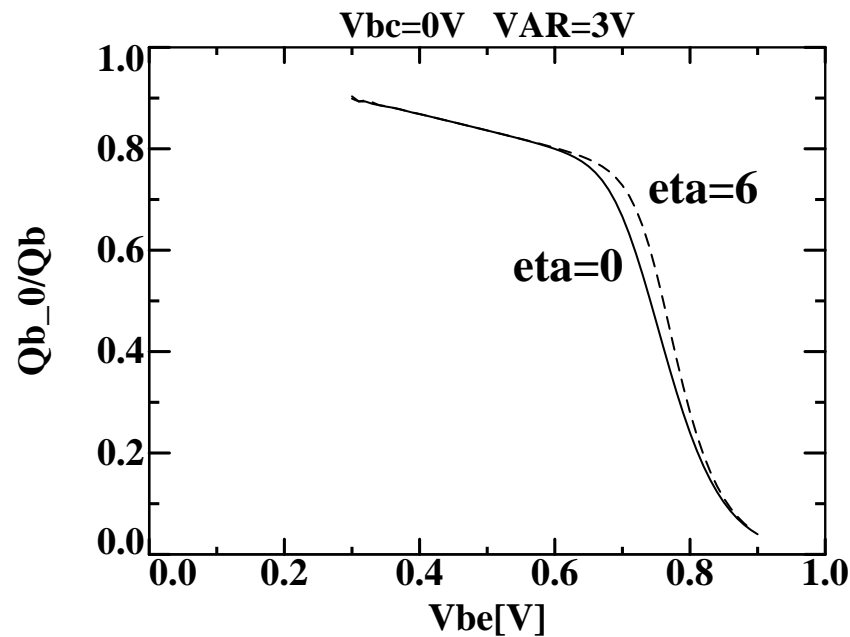
q_1

Early effect

⇓

q_2

high injection base



Main differences comparing with GP:

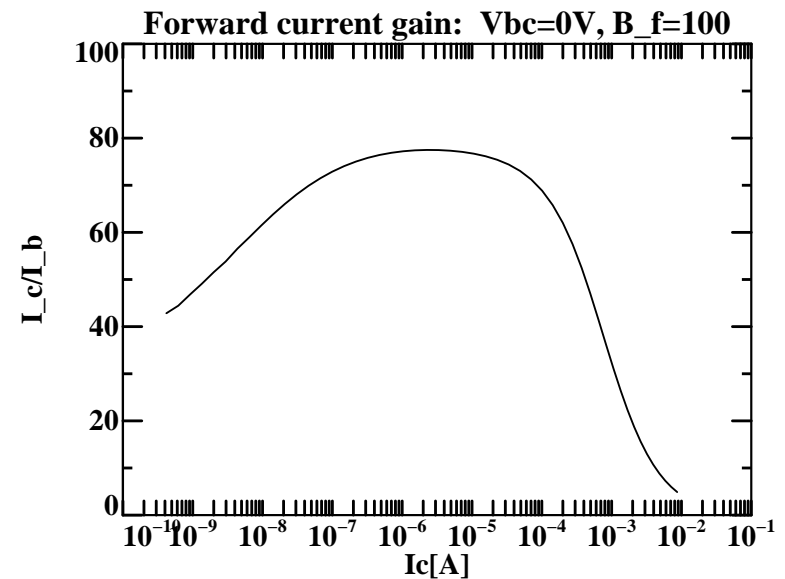
- bias dependent Early effect.
- punch-through: $Q_{T_e} + Q_{T_c} = -Q_{b_0}$
- Mextram has only one knee current
- In Mextram high injection is related to the diffusion charges

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forward active region: ideal bottom/sidewall component +
non-ideal component based on SRH recombination

$$I_b = I_{b_f} \cdot \frac{\exp(V_{b2e1}/V_t) - 1}{\exp[V_{b2e1}/(2 \cdot V_t)] + \exp[V_{L_f}/(2 \cdot V_t)]} + \frac{I_s}{B_f} \cdot \left[\exp\left(\frac{V_{b2e1}}{V_t}\right) - 1 \right] \cdot (1 - X I_{b_i}) + \frac{I_s}{B_f} \cdot \left[\exp\left(\frac{V_{b1e1}}{V_t}\right) - 1 \right] \cdot X I_{b_i}$$

Parameters: B_f , $X I_{b_i}$, I_{b_f} , V_{L_f}



reverse base current contains three components: $I_b = I_{ex} + I_{b3} + I_{sub}$

- electrons injected from the collector into the extrinsic base: I_{ex}
- non-ideal component based on SRH recombination: I_{b3}
- substrate current of the parasitic PNP: I_{sub}

$$I_{ex} = \frac{I_s}{B_{ri}} \cdot \frac{\exp\left(\frac{V_{b1c1}}{V_t}\right) - 1}{1 + q_{ex}}$$

$$I_{b3} = I_{br} \cdot \frac{\exp(V_{b1c1}/V_t) - 1}{\exp[V_{b1c1}/(2 \cdot V_t)] + \exp[V_{Lr}/(2 \cdot V_t)]}$$

$$I_{sub} = \frac{2 \cdot I_{ss} \cdot \left\{ \exp\left(\frac{V_{b1c1}}{V_t}\right) - 1 \right\}}{1 + \sqrt{1 + 4 \cdot \frac{I_s}{I_{ks}} \cdot \left\{ \exp\left(\frac{V_{b1c1}}{V_t}\right) - 1 \right\}}}$$

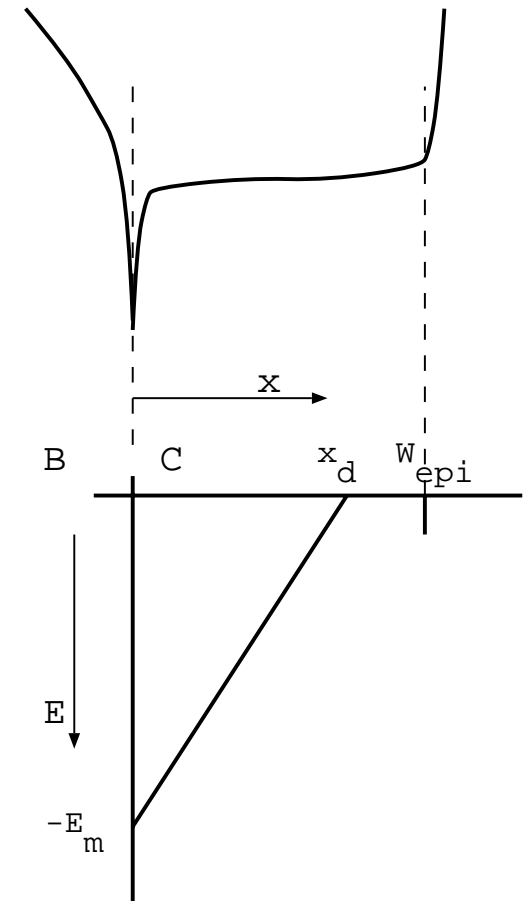
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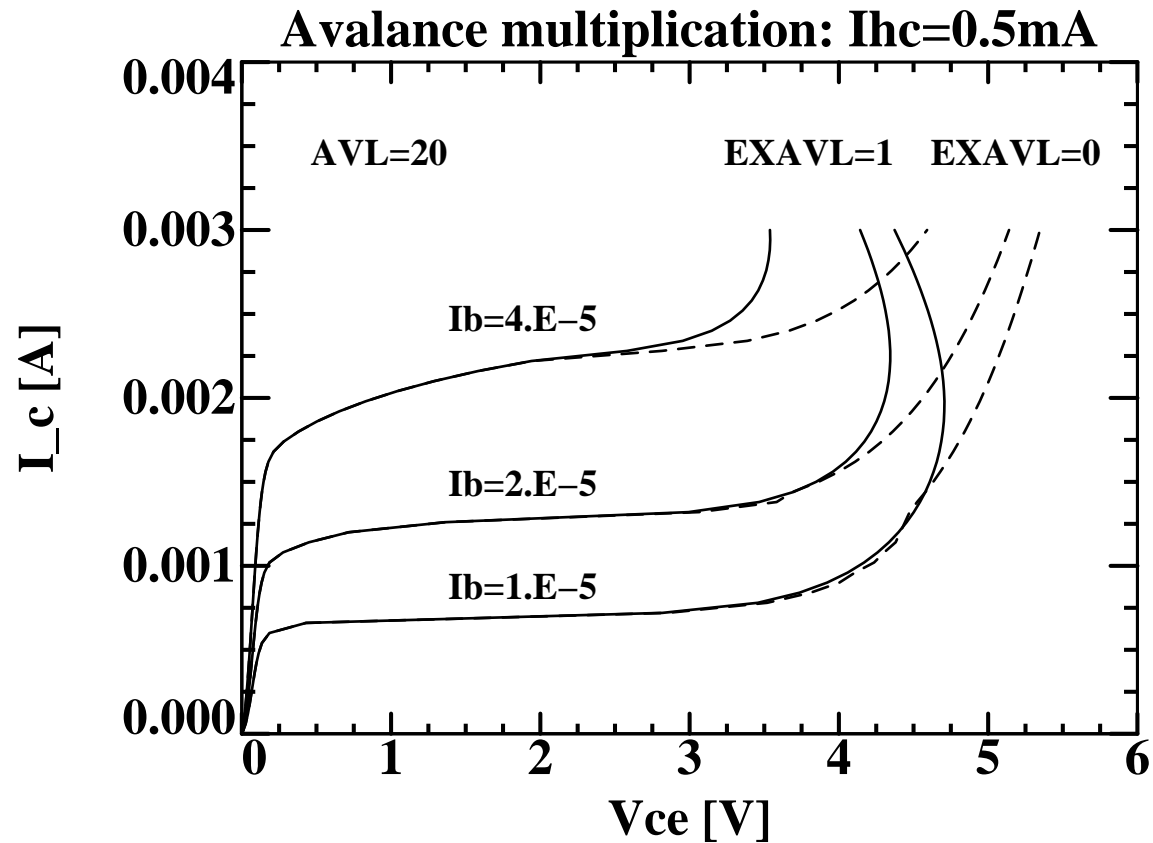
Weak avalanche model means: $I_{avl} < I_c \approx I_n$

$$I_{avl} = I_n \cdot \int_0^{x_d} \alpha_n \cdot \exp\left(\frac{-b_n}{|E(x)|}\right) \cdot dx$$

$$|E(x)| = E_m \cdot \left(1 - \frac{x}{\lambda}\right) \approx E_m \cdot \frac{1}{1 + \frac{x}{\lambda}}$$

x_d , E_m and λ are bias-dependent and can all be related to the depletion layer thickness at zero bias (x_{d0}).





Parameters: $Avl = b_n \cdot x_{d0}$, (I_{hc} , SFH , XP )

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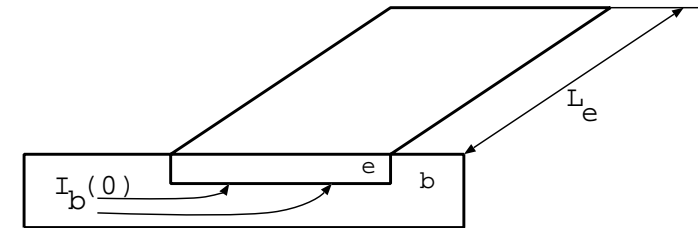
- 1) R_e, R_{b_c}, R_{c_c} are constant
- 2) R_{b_v}, R_{c_v} are bias-dependent

Resistance model R_b of the pinched base accounts for

- base charge modulation: $R_b = R_{b_v}/q_b$
- DC current crowding, based on elementary model (Hauser)

A very good approximation (Groendijk) is;

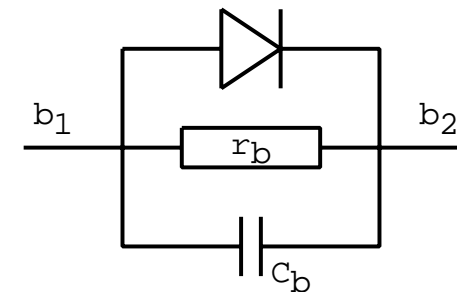
$$I_{b1b2} = \frac{2 \cdot V_t}{3 \cdot R_b} \cdot \left(\exp\left(\frac{V_{b1b2}}{V_t}\right) - 1 \right) + \frac{V_{b1b2}}{3 \cdot R_b}$$



note: with reversal base current (decharging, avalanche) $r_b > R_{b_v}$!!

- AC current crowding

$$C_b = \frac{1}{5} \cdot \left(\frac{\partial Q_{Te}}{\partial V_{b2e1}} + \frac{\partial Q_{be}}{\partial V_{b2e1}} + \frac{\partial Q_n}{\partial V_{b2e1}} \right)$$



Collector epilayer resistance model

forces the voltage on node $c_2 \rightarrow I_r, Q_{bc}, Q_{epi}, Q_{Tc}$

In this way we model base push out and the collector transit time.

Two cases have to be considered:

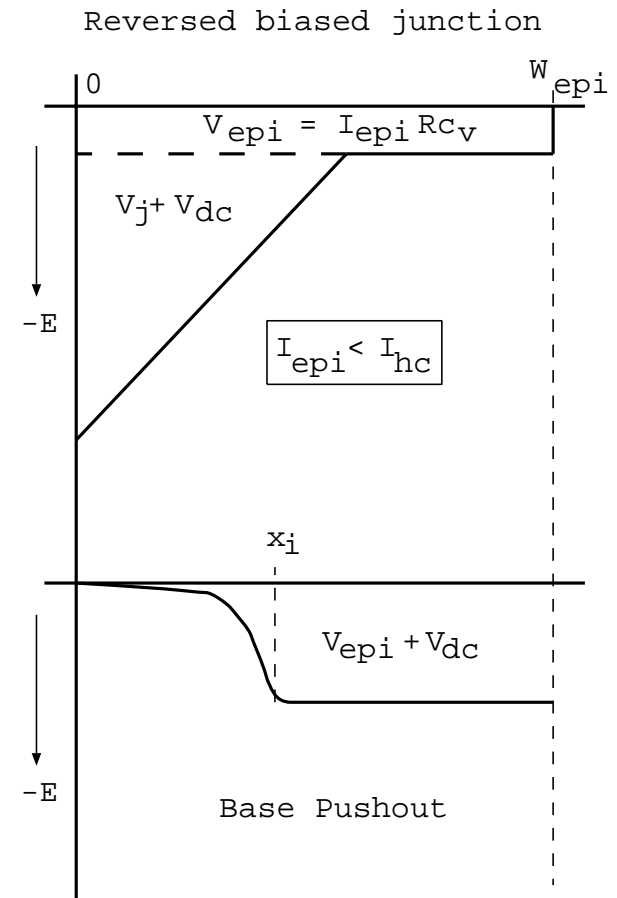
- ohmic current flow: model of Kull;
neutrality $p + N_{epi} \approx n, J_p = 0$
- velocity saturated current flow

$$I_{epi} = \frac{E_c + V_{epi}}{R_{cv}}$$

$$E_c = V_t \cdot \left[K_0 - K_w - \ln \left(\frac{K_0 + 1}{K_w + 1} \right) \right]$$

$$K_0 = \sqrt{1 + 4 \cdot \exp[(V_{b2c2} - V_{dc})/V_t]}$$

$$K_w = \sqrt{1 + 4 \cdot \exp[(V_{b2c1} - V_{dc})/V_t]}$$



Combine ohmic and saturated current flow and include injection region:

$$I_{hc} \longrightarrow I_{low}$$

$$R_{cv} \longrightarrow R_{cv} \cdot (1 - X_i/W_{epi})$$

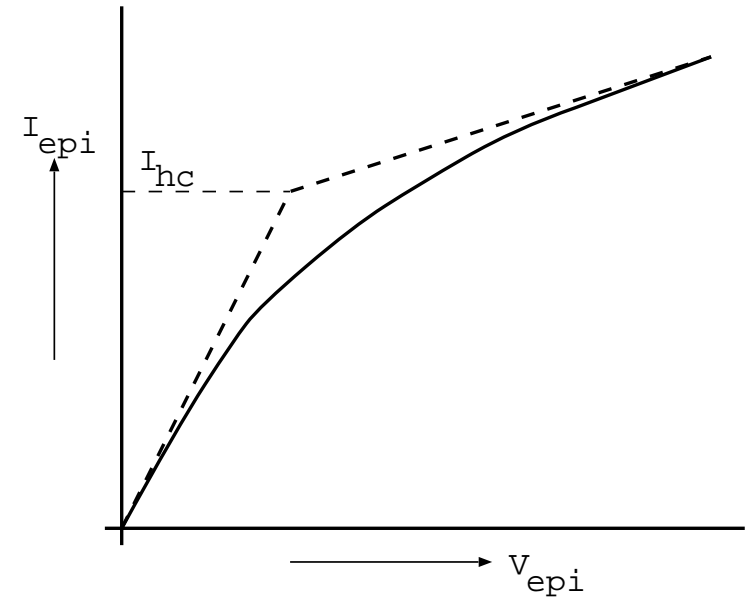
$$SCR_{cv} \longrightarrow SCR_{cv} \cdot (1 - X_i/W_{epi})^2$$

Final epilayer resistance model:

$$X_i/W_{epi} = \frac{E_c}{I_{epi} \cdot R_{cv}}$$

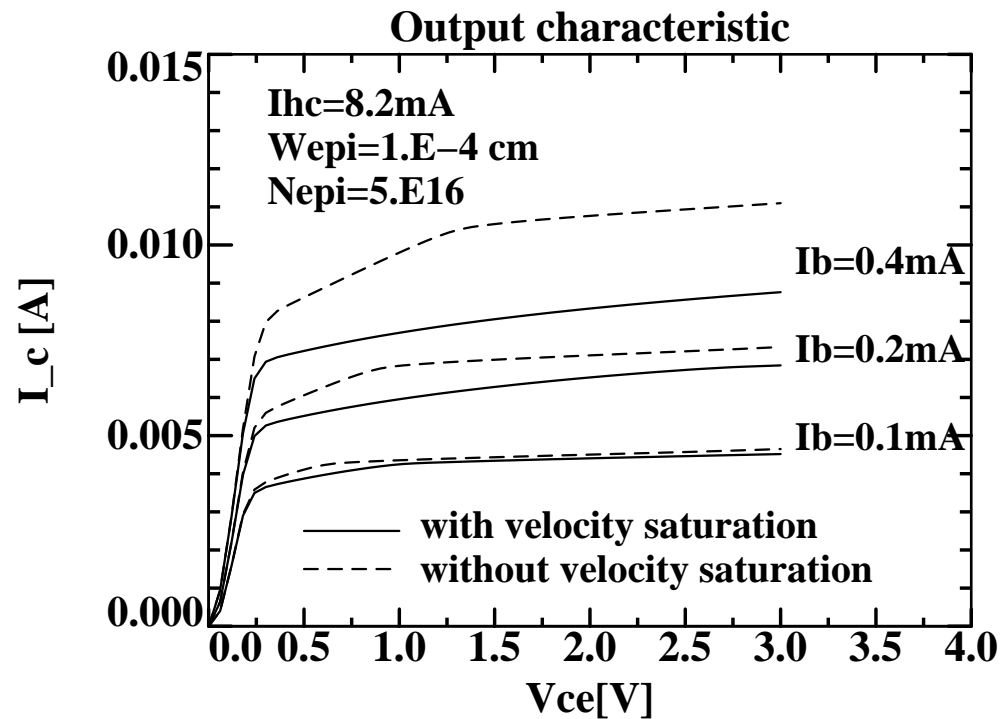
$$I_{low} = \frac{I_{hc} \cdot V_{epi}}{V_{epi} + I_{hc} \cdot R_{cv} \cdot (1 - X_i/W_{epi})}$$

$$I_{epi} = I_{low} + \frac{V_{epi} - I_{low} \cdot R_{cv} \cdot (1 - X_i/W_{epi})}{SCR_{cv} \cdot (1 - X_i/W_{epi})^2}$$



This leads to a cubic equation for I_{epi}

Influence of velocity saturation on the output characteristic.



Parameters: R_{c_v} , I_{hc} , SCR_{c_v} , V_{dc}

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Noise Sources

1. thermal noise;

$\overline{i_N^2} = \frac{4 \cdot k \cdot T}{R}$ is valid for R_e , R_{c_c} , R_{b_c} and slightly different for R_{b_v} .

2. shot noise;

$\overline{i_N^2} = 2 \cdot q \cdot I$ is valid for I_n , I_{b_1} , $I_{b_1}^s$, I_{b_2} , I_{b_3} , I_{ex} and XI_{ex}

3. $1/f$ noise of the base current;

$\overline{i_N^2} = KF \cdot I_b^{AF}$ is valid for I_{b_1} , $I_{b_1}^s$, I_{b_3} , I_{ex} , XI_{ex}

and for the non-ideal forward base current we have $KFN \cdot (I_{b_2})^2$

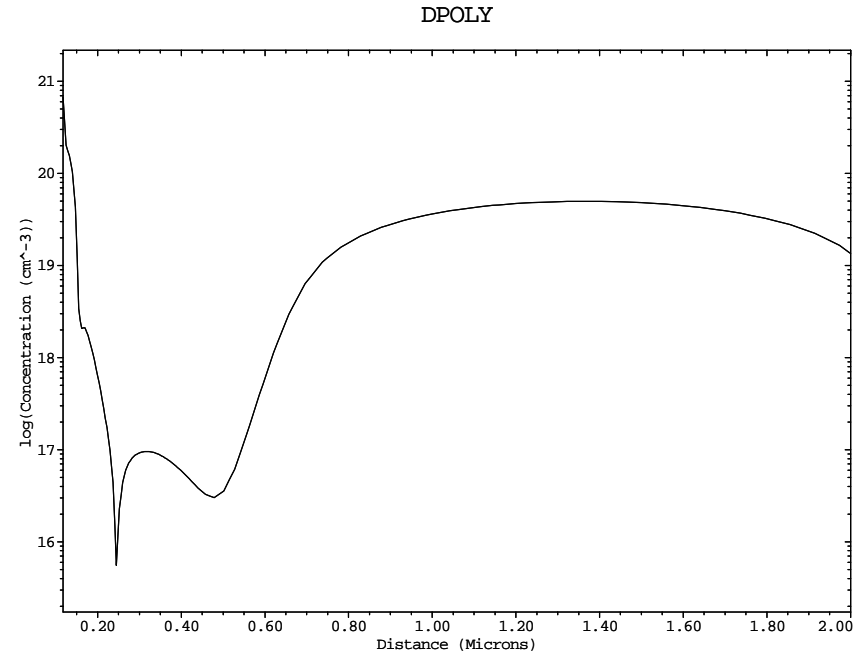
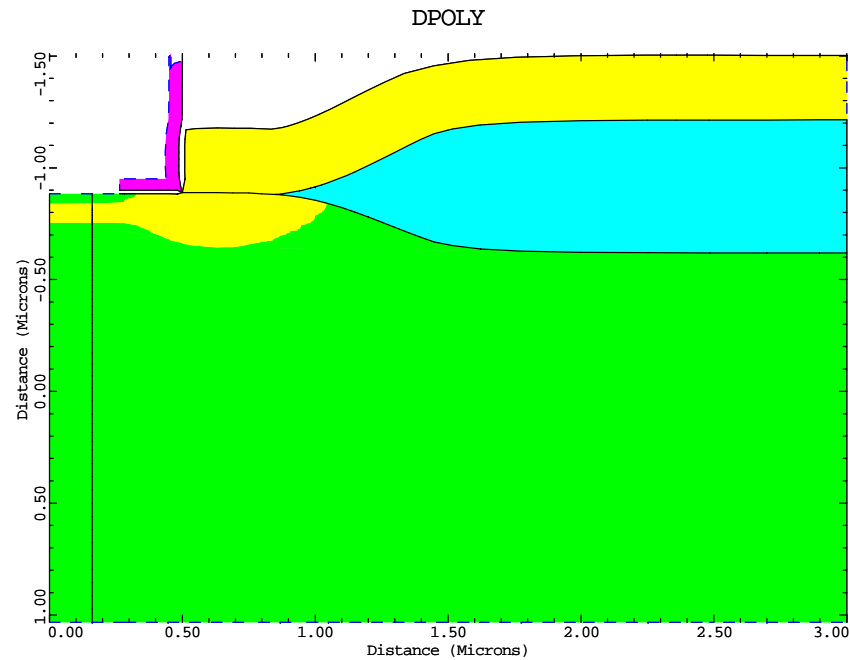
The Mextram model contains extensive temperature scaling rules. They are applied to the following parameters:

| | |
|---------------------|--|
| Resistances | $R_{bc}, R_{bv}, R_{cv}, R_{cc}$ |
| Capacitances | $C_{je}, V_{de}, C_{jc}, V_{dc}, XP, C_{js}, V_{ds}, Q_{b0}, Q_{n0}, m_{\tau}$ |
| Saturation currents | I_s, I_{s_s} |
| gain modelling | $B_f, I_{bf}, V_{Lf}, B_{ri}, I_{br}, V_{Lr}, I_k, I_{k_s}$ |
| avalanche | Avl |

The parameters in the temperature scaling rules are:

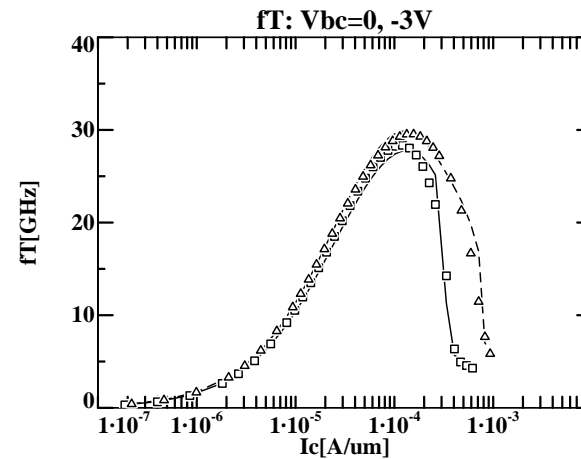
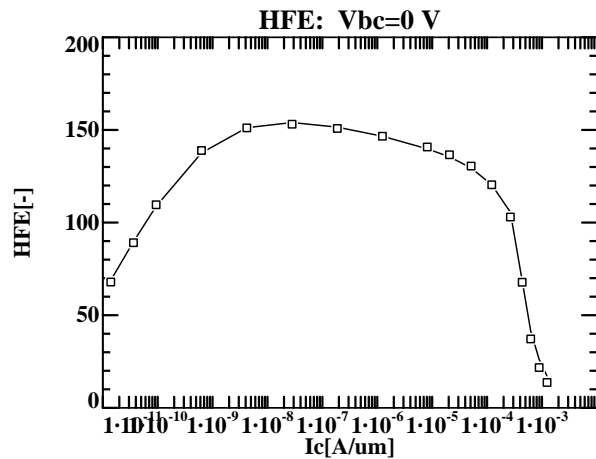
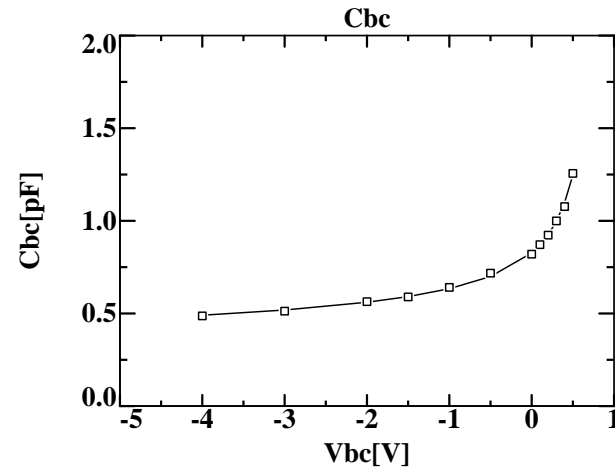
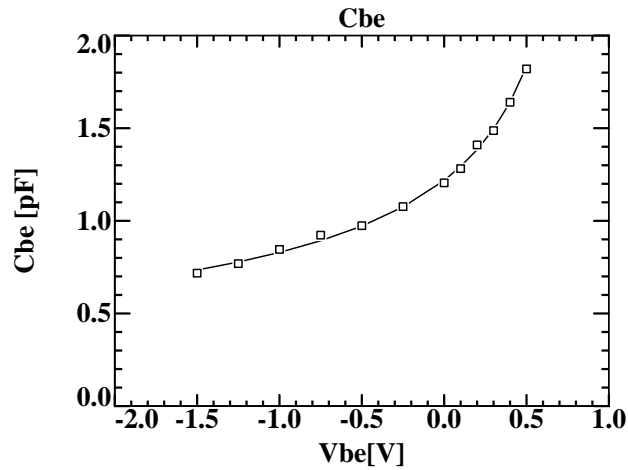
| | |
|--|--------------------|
| $V_{ge}, V_{gb}, V_{gc}, V_{gs}, V_{gj}$ | Bandgap voltages |
| $A_b, A_{epi}, A_{ex}, A_c, A_s$ | Mobility exponents |
| V_i, N_a | Q_{b0} |
| E_r | V_{Lf}, V_{Lr} |

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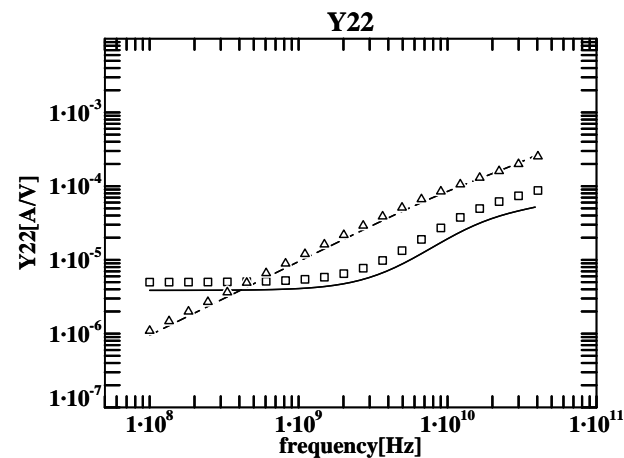
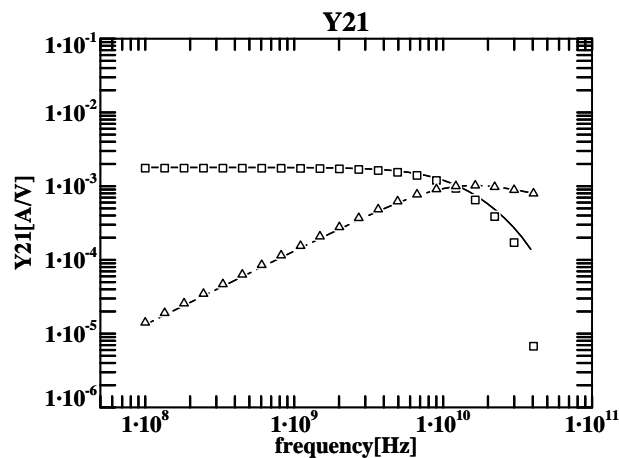
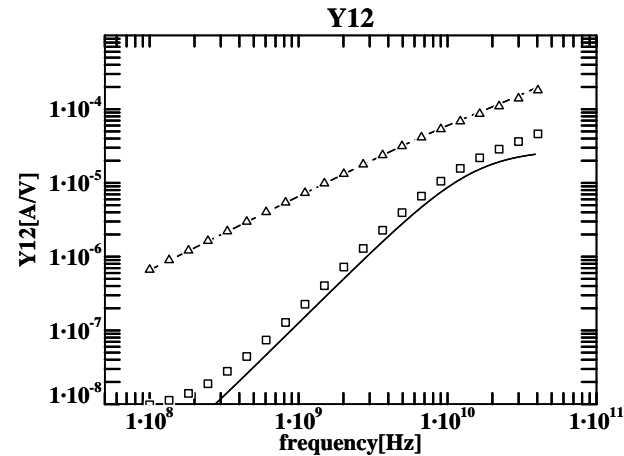
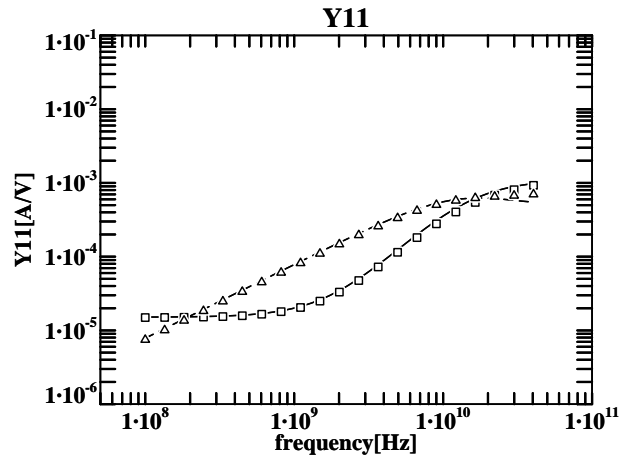


Cross section of the double poly bipolar transistor and the doping profile at the center of the device.

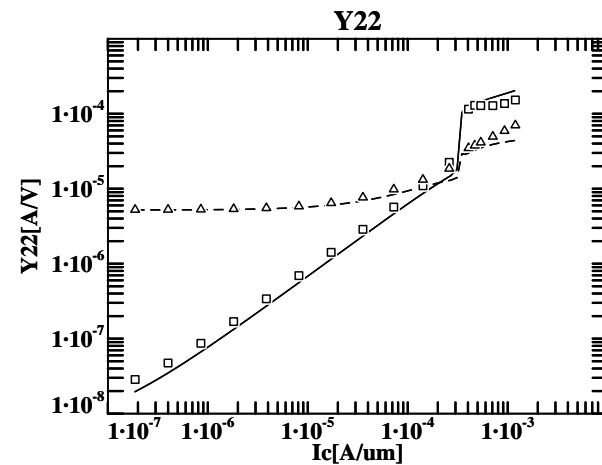
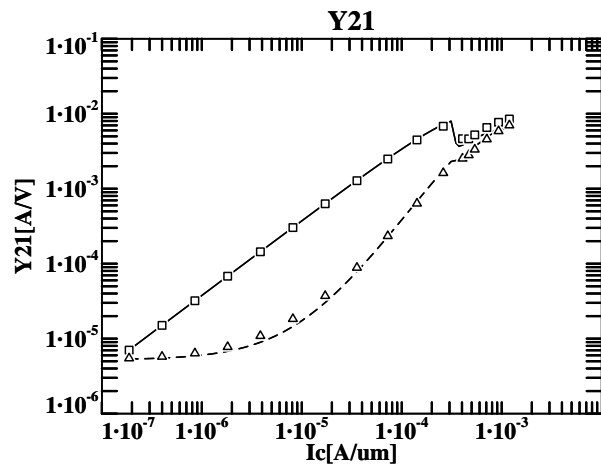
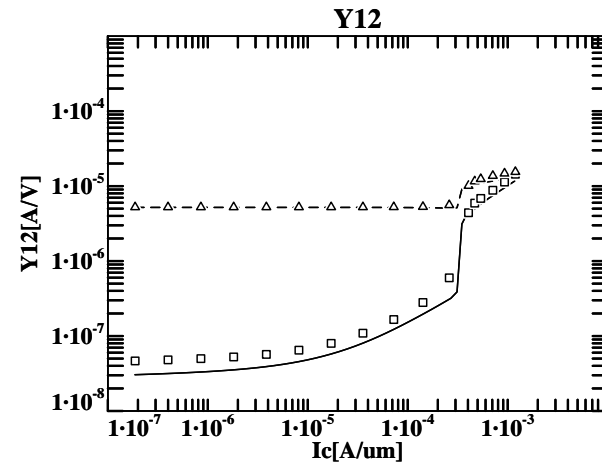
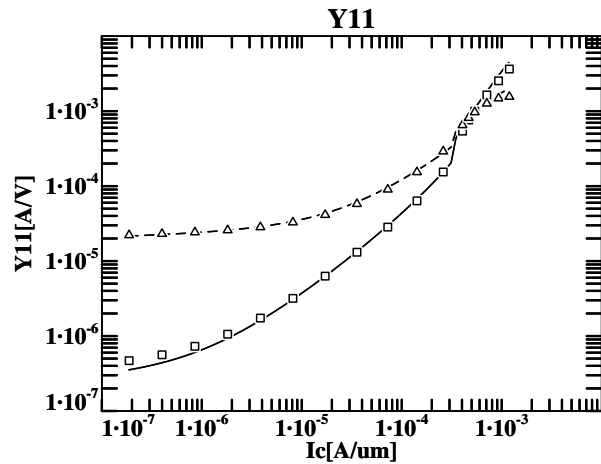
dpoly1: Results parameter extraction Mextram model versus Medici



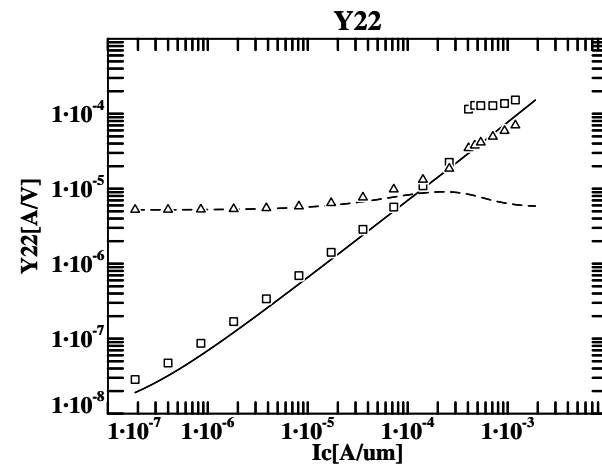
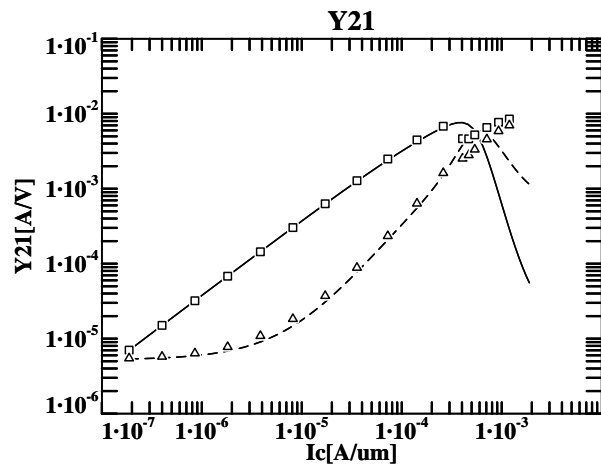
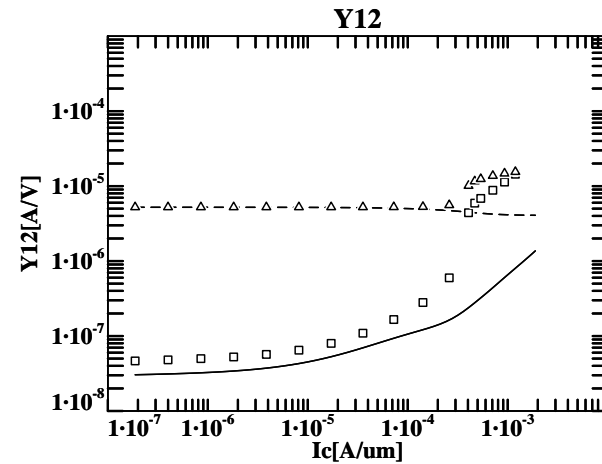
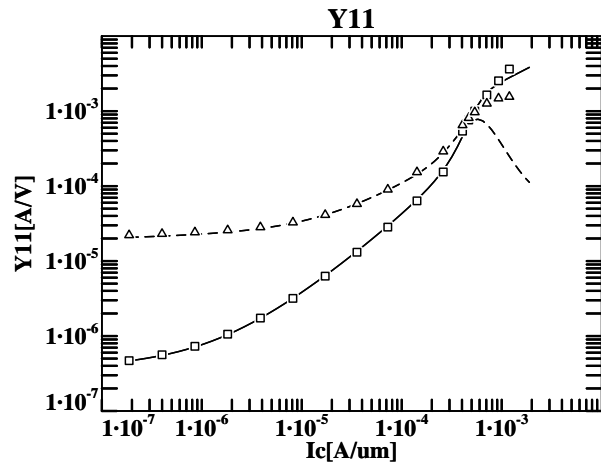
dpoly1: bias: Vbe=0.85 Vce=0.5 Medici Mextram



dpoly1: bias $V_{bc}=0.0V$ freq: 1.0Ghz Medici Mextram



dpoly1: bias $V_{bc}=0.0V$ freq: 1.0Ghz Medici SGP



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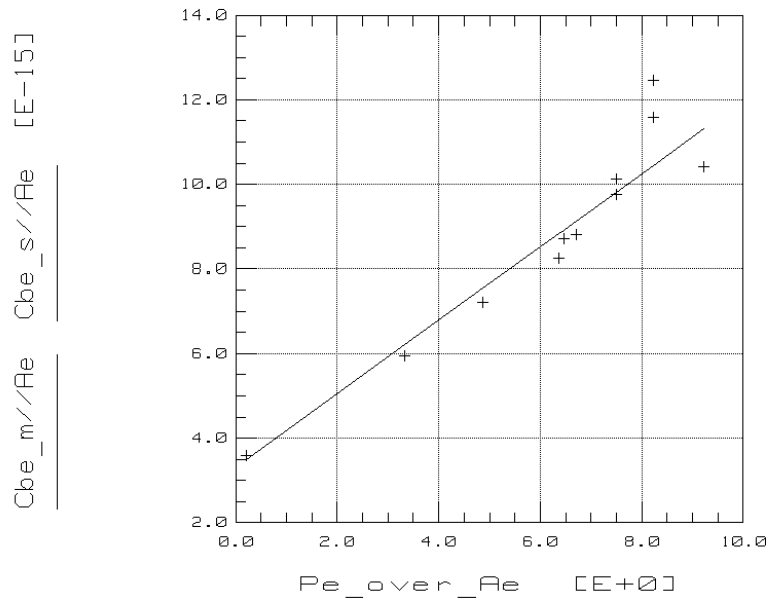
In general we have as a geometrical scaling rule:

- rectangular geometry $W \times L$
- model parameter $p = p_b \cdot W \cdot L + 2 \cdot p_s \cdot (W + L)$
No corner contributions.
- p_b and p_s are unity parameters and process-dependent
They are correlated with PCM data like ρ_{ext} , ρ_{pinch} and BV_{cb0}

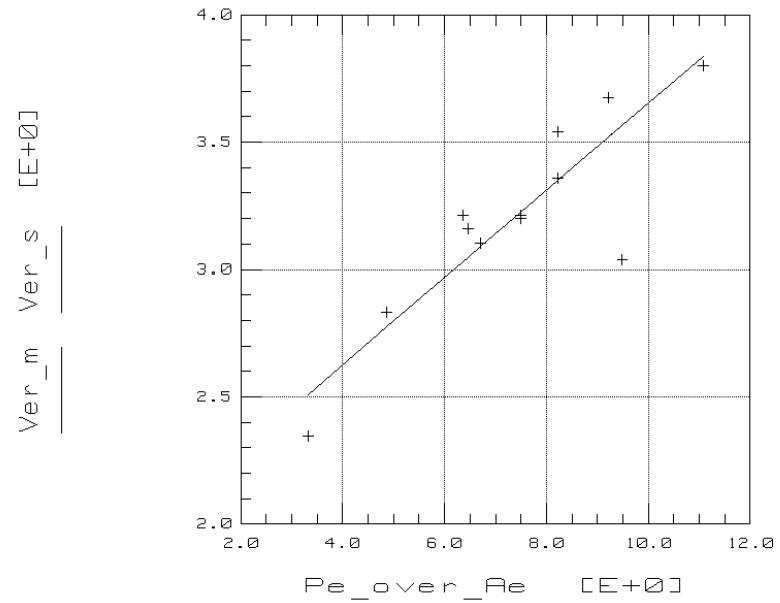
Statistical models can use the same scaling rules !

Average values \bar{p} and variances σ_p^2 follow from \bar{W} , \bar{L} , \bar{p}_b , \bar{p}_s
and σ_W^2 , σ_L^2 , $\sigma_{p_b}^2$, $\sigma_{p_s}^2$.

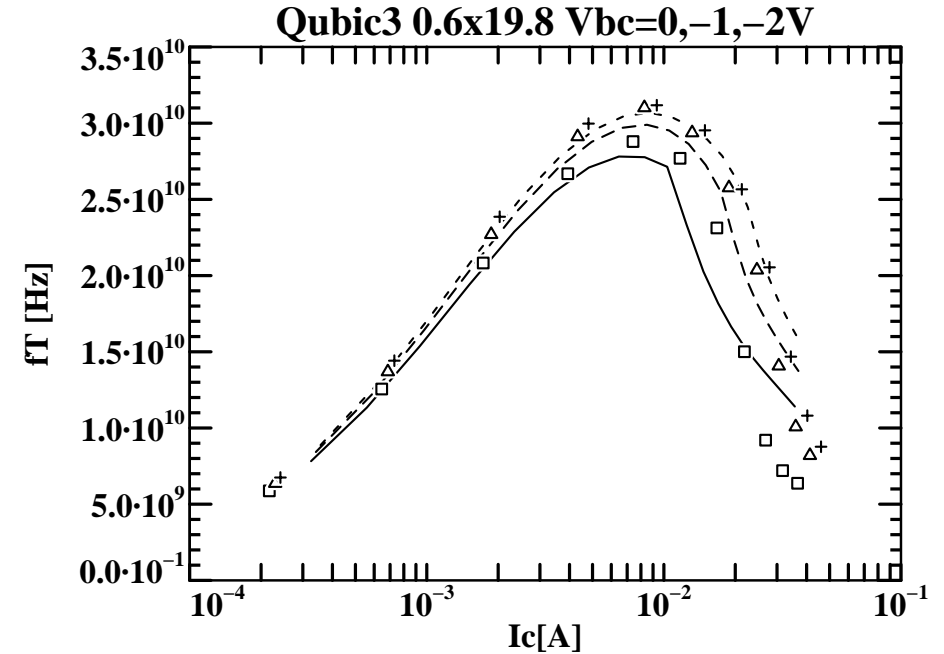
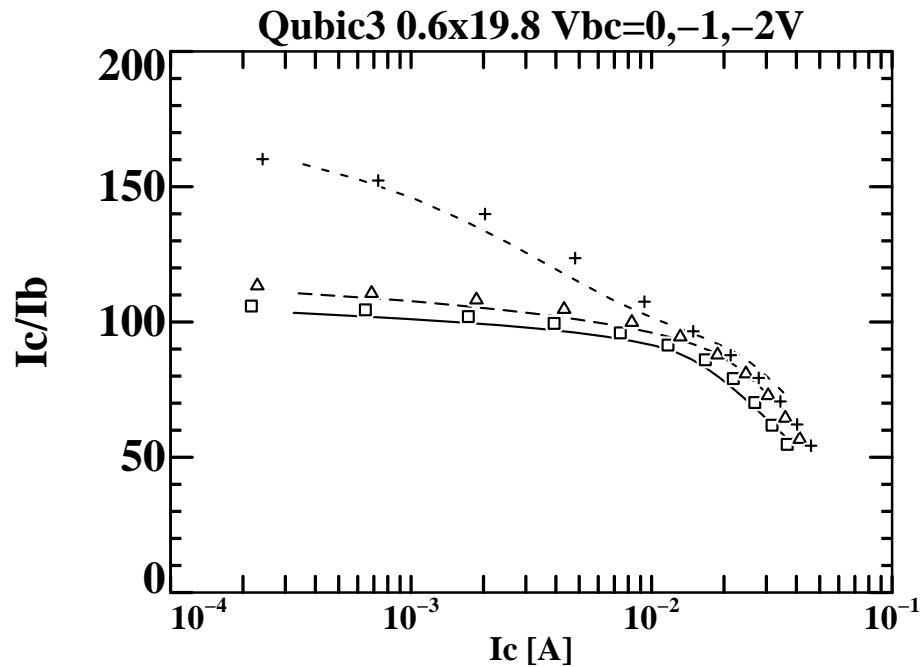
Plot geom_mxt503/parameters/Cbe/Cbe (On



Plot geom_mxt503/parameters/Av1_Early/Ver (On)



Geometry dependence of the base-emitter capacitance and the reverse Early voltage.



Device characteristic after geometric scaling of a double-poly npn transistor, fabricated in the Philips Semiconductors Qubic-3 process.

Emitter dimension in silicon : $0.6 \times 19.8 \mu\text{m}^2$.

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- From benchmarking of Mextram versus Vbic.
 - more physics based (complicated) model description
 - less number of parameters
 - more correlations between DC and AC
 - more experience needed to perform parameter extraction
 - only one free parameters to model cut off frequency f_T
 - no self heating, because in Philips:
 - * feature of the inhouse circuit simulator Pstar
 - * available for all device models.
 - * not only self heating, also from neighbours is easy possible
 - no substrate series resistance

What can be expected of Mextram 504?

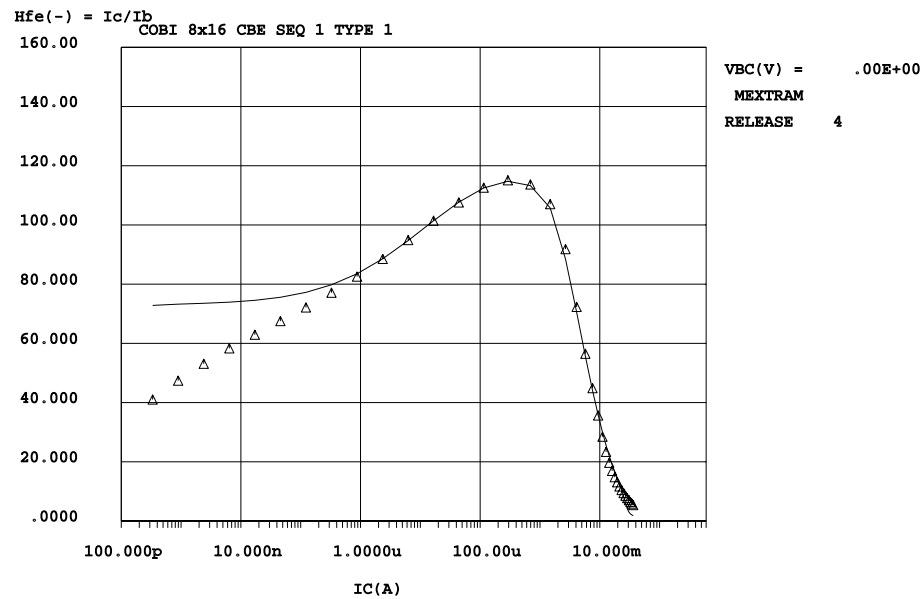
- review of the complete model → documentation
- introduction of additional parameters in existing Mextram model formulations
- modelling of SiGe transistors
- until now no new phenomenon like collector breakdown, tunneling/avalanche at BE junction

Achieved until now:

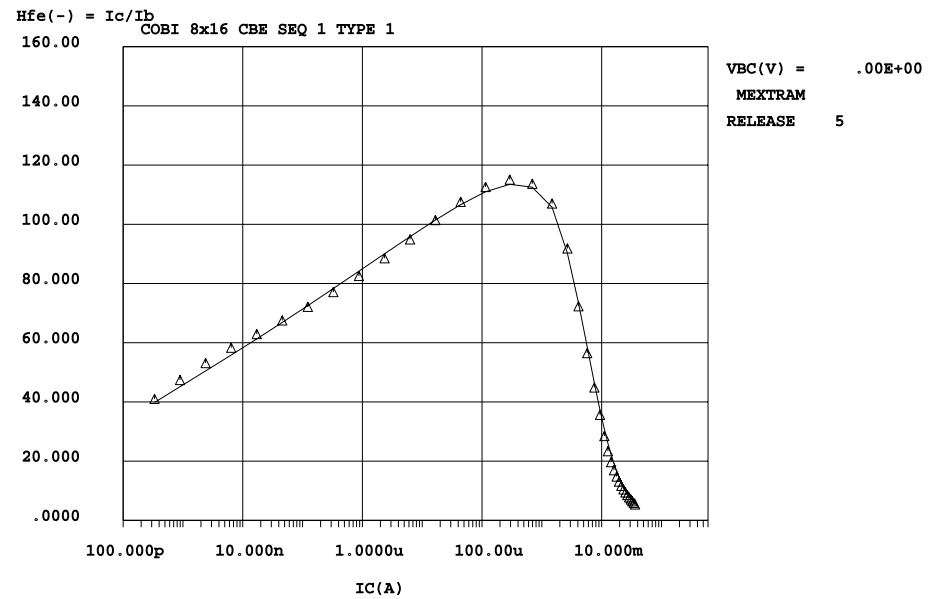
- more flexible non-ideal forward base current, compatible with GP and Mextram 503.2 (one additional parameter).
- modelling of the SiGe transistor (two additional parameters)
- simplified and improved thermal noise model for the bias dependent part of the base resistance Rb_v (DC current crowding)
- improved temperature scaling of neutral base charge Q_{b_0} (Early effect).

example: COBI process, CBE layout

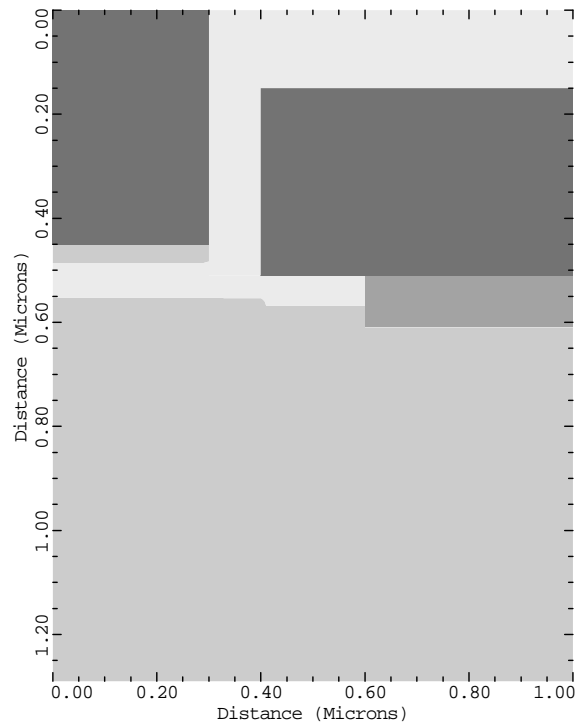
Mextram 503.2



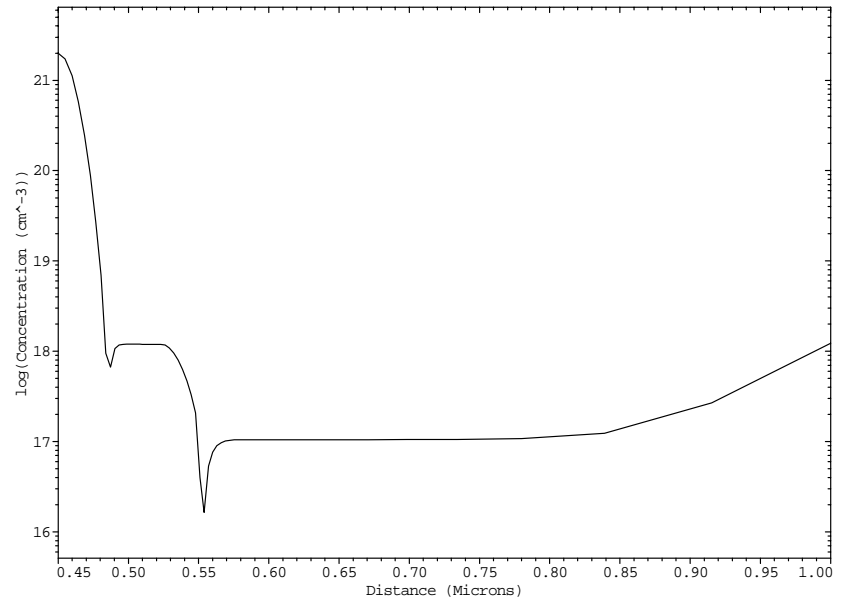
Mextram 504.0



Selectively grown Si/SiGe HBTs with 2 basecontacts

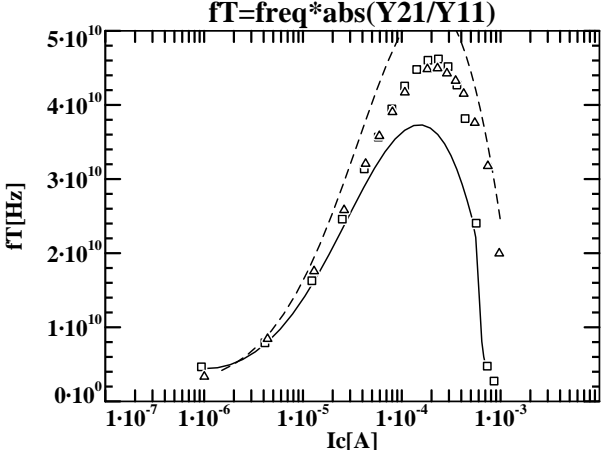
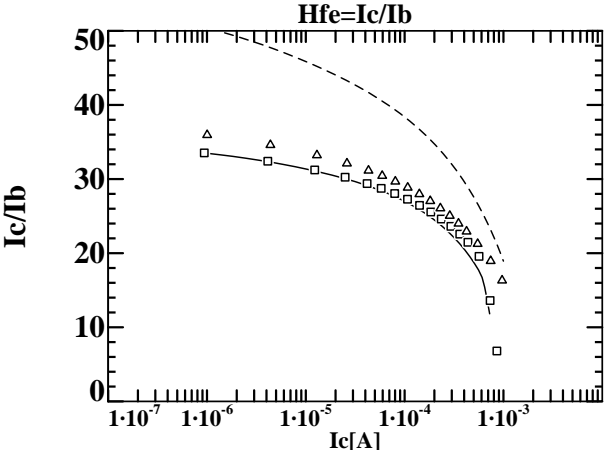
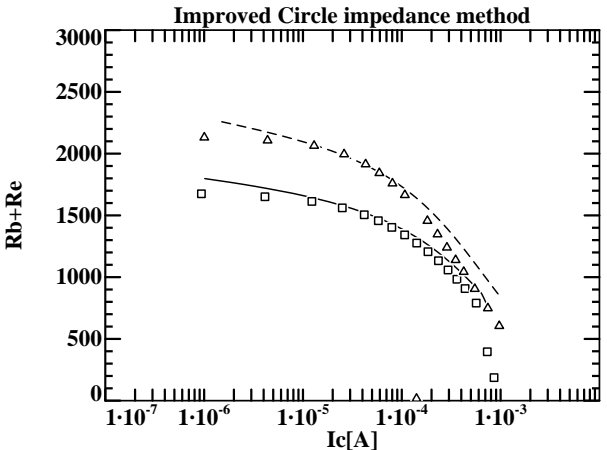
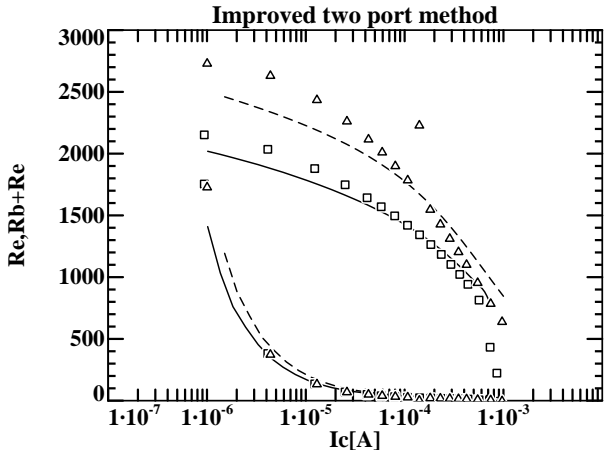


Whitney selectively grown Si/SiGe HBTs with 2 basecontacts



Cross section of the SiGe transistor and the doping profile at the center of the device.

SiGe freq=10.0Ghz, Vbc= 0,-3V XCjc=0.5 Medici



Dependence of collector voltage on current gain and cut-off frequency f_T is different for SiGe transistors.

Moll-Ross relation for the main electron current

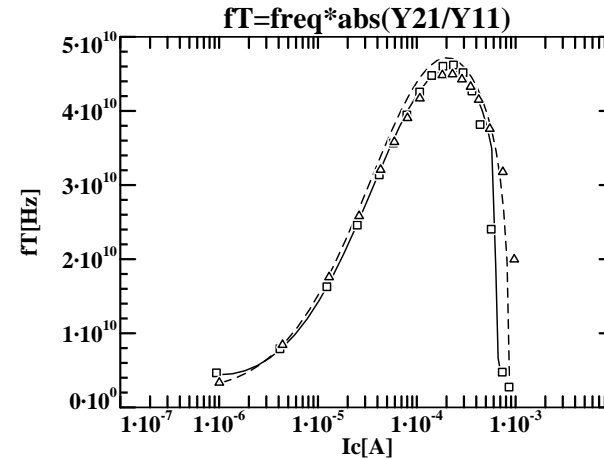
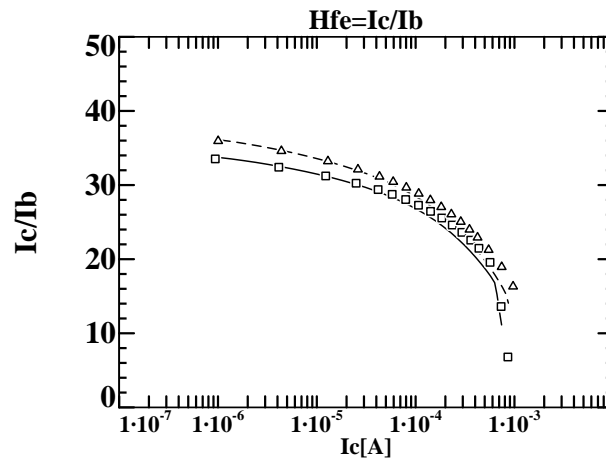
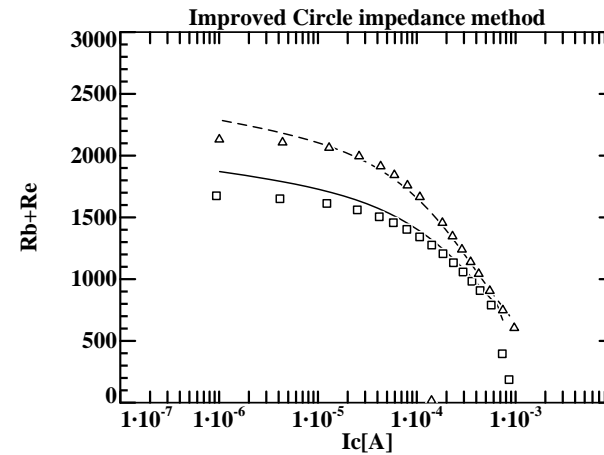
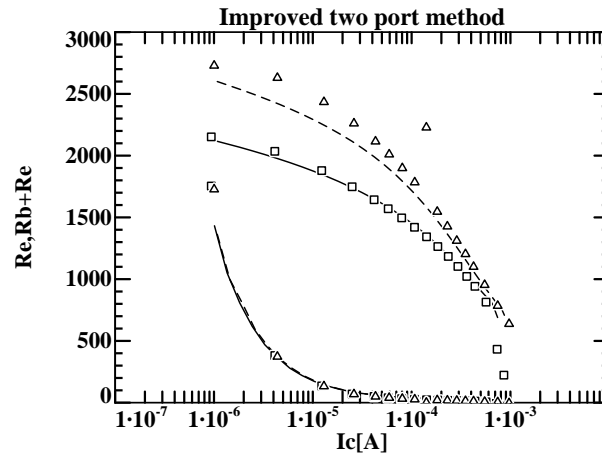
$$I_n = I_s \cdot \left[\exp\left(\frac{V_{b2e1}}{V_t}\right) - \exp\left(\frac{V_{b2c2}}{V_t}\right) \right] \cdot \frac{Q_{b0}}{Q_{b0} + Q_{T_e} + Q_{T_c} + Q_{be} + Q_{bc}}$$

$$I_n = \frac{I_f - I_r}{q_b^i} \quad q_b^i = 1 + \frac{Q_{be} + Q_{bc}}{Q_{b0}} + \frac{V_{T_e}}{VER} + \frac{V_{T_c}}{VEF}$$

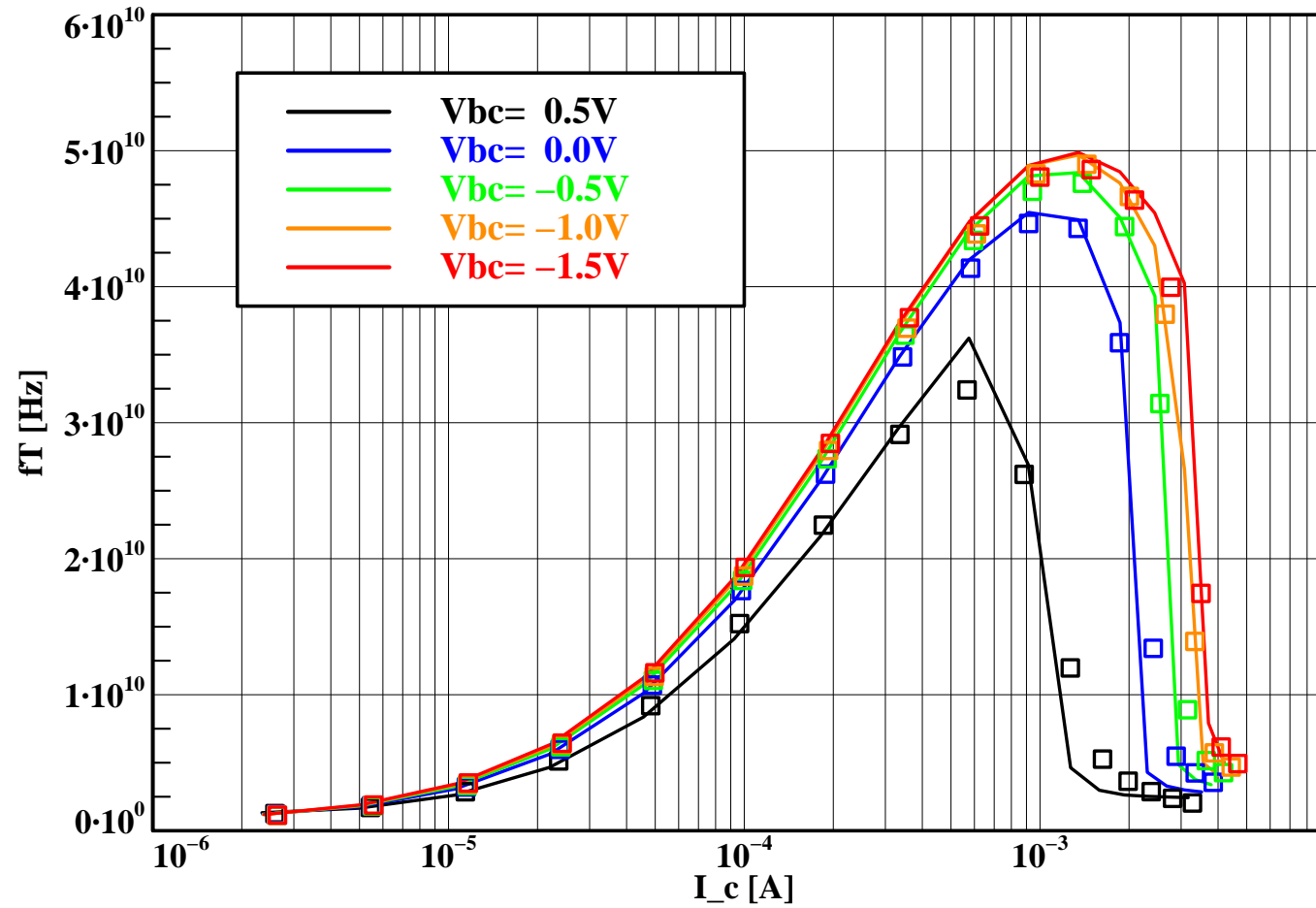
$$R_b = \frac{R_{bv}}{q_b^r} \quad q_b^r = 1 + \frac{Q_{be} + Q_{bc}}{Q_{b0}} + \frac{Q_{T_e} + Q_{T_c}}{Q_{b0}}$$

$$VER = \frac{Q_{b0}}{(1 - XC_{je}) \cdot C_{je}} \quad VEF = \frac{Q_{b0}}{XC_{jc} \cdot C_{jc}}$$

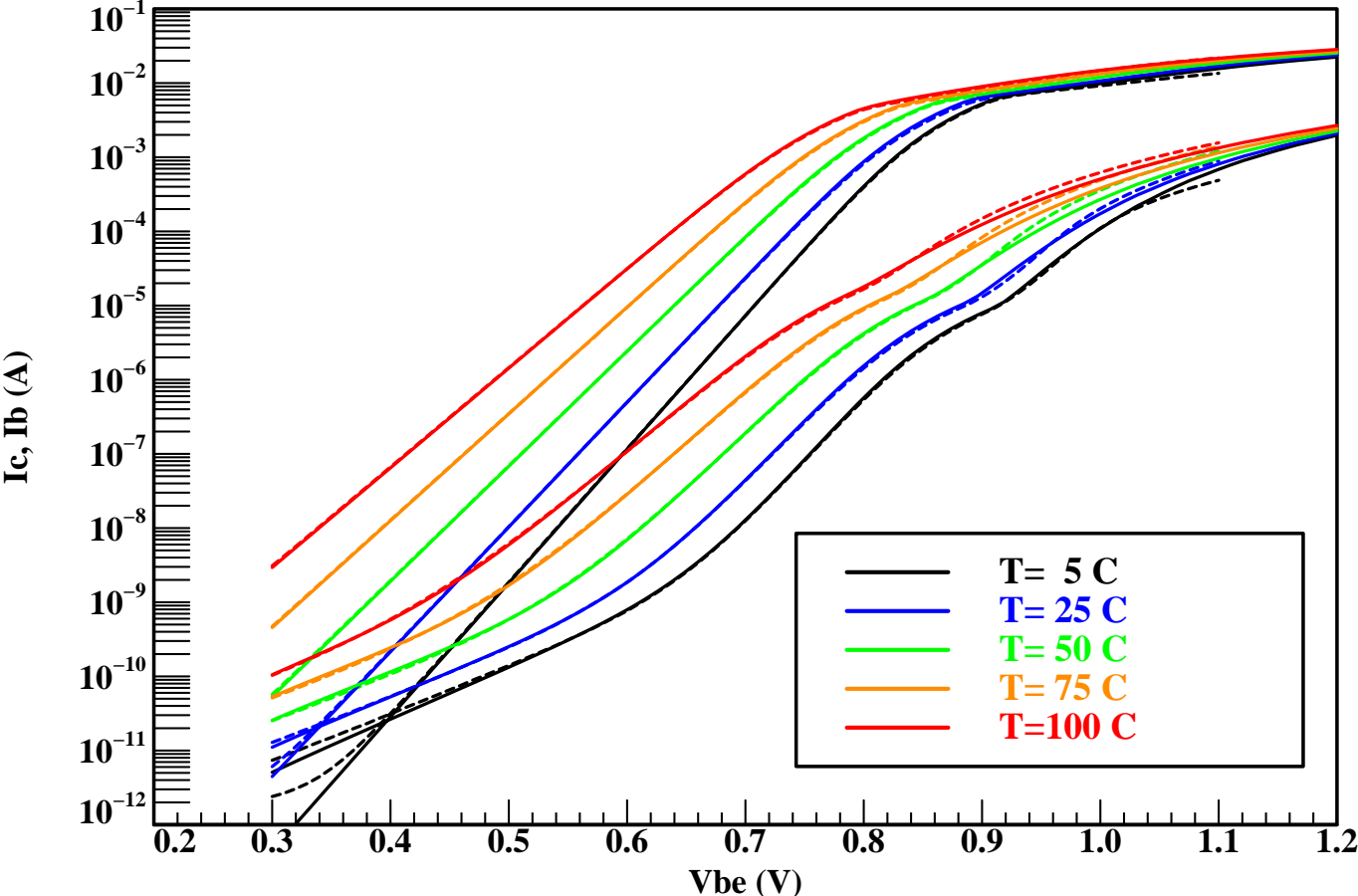
SiGe freq=10.0Ghz, Vbc= 0,-3V Medici M504



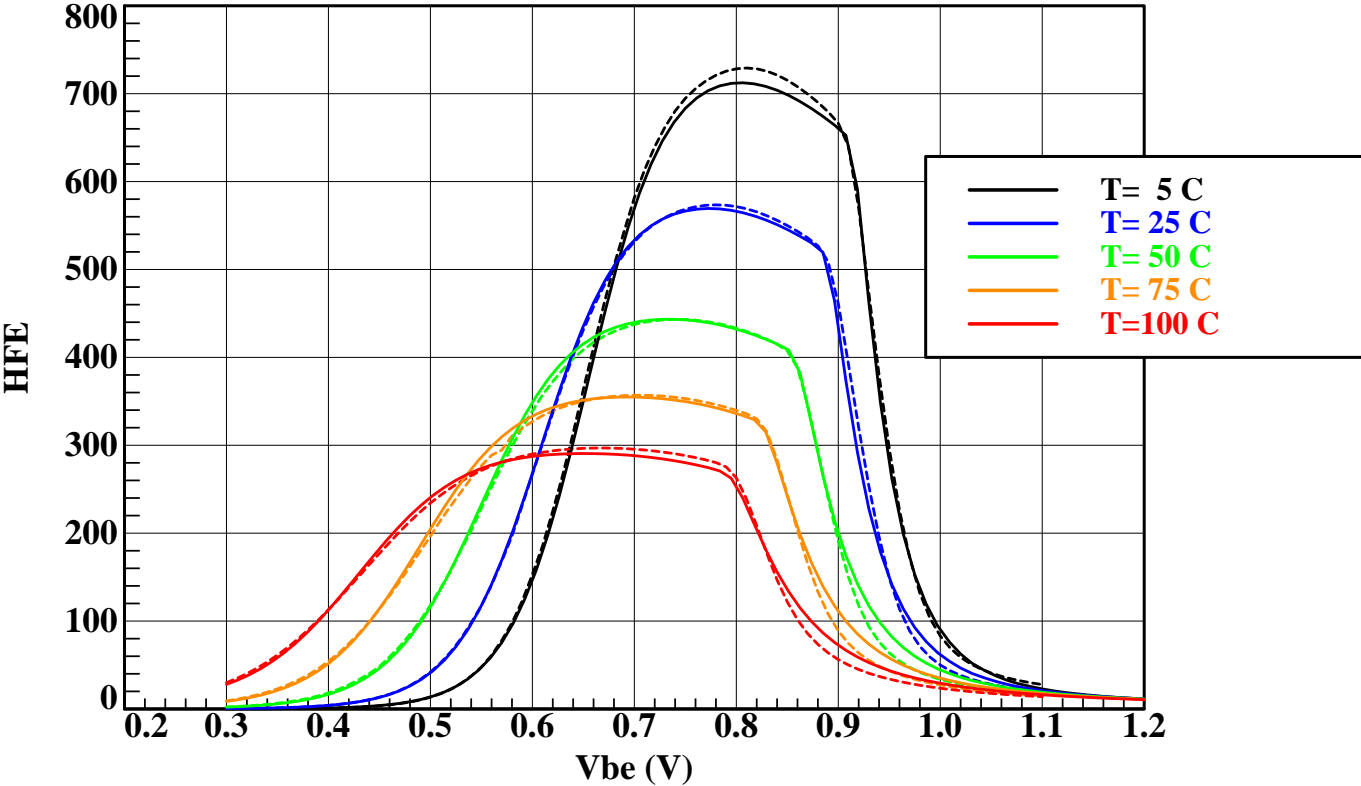
SiGe Measured/Mextram_504.0



SiGe Measured/Mextram_504 Gummel plot: Vbc=0V



MeasuredMextram_504 HFE plot: SiGe transistor



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- Bias-dependent Early effect modelled by means of depletion charges
- Charge storage in quasi-neutral regions modelled independently from currents
- Influence of built-in electric field
- High-injection effects in the base
- Current- and voltage dependent c-b depletion capacitance
- Quasi-saturation and hot-carriers (Kirk) effect in the collector epilayer
- Bias-dependent current gain
- Avalanche multiplication valid for all collector current levels
- Conductivity modulation of the base
- DC and AC e-b current crowding
- Sidewall contributions to currents and depletion charges
- Explicit modelling of inactive regions: parasitic PNP, split c-b capacitance and substrate effects
- Temperature dependencies of the model parameters
- Physics-based parameters, facilitating geometrical scaling and statistical modelling

- Mextram 504 is able to model SiGe transistors accurately
 - Bias dependency of the collector current.
 - Cut off frequency f_T and small signal Y parameters.
 - Temperature scaling.
- More flexibility in parameter extraction by introducing additional parameters.

Future work:

- Temperature scaling in general and in particular of f_T