MOS MODEL 9

from Physics to Equations

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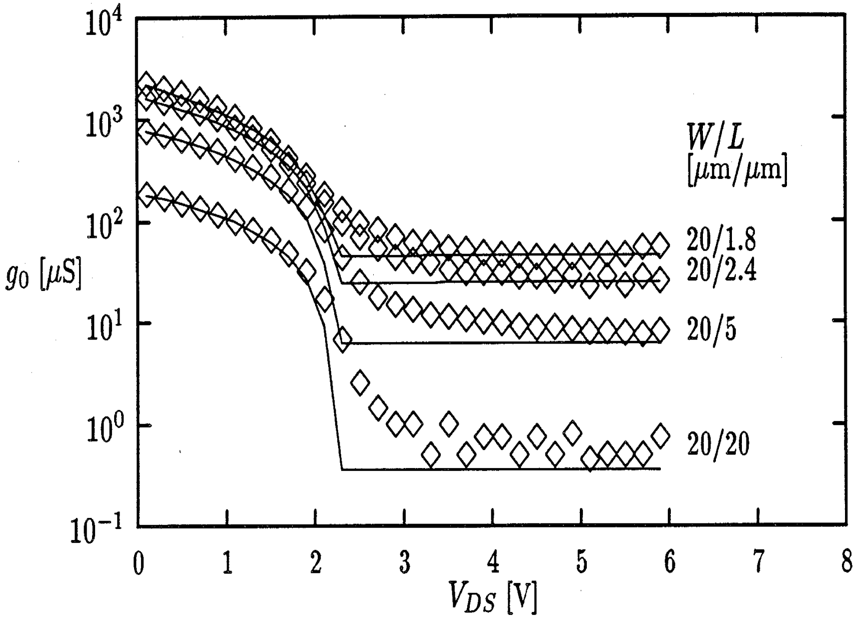
contents

introduction

- MOS MODEL 9 has been introduced in 1990 on C3DM
- before 1990 MOS MODEL 7 was the most advanced compact model

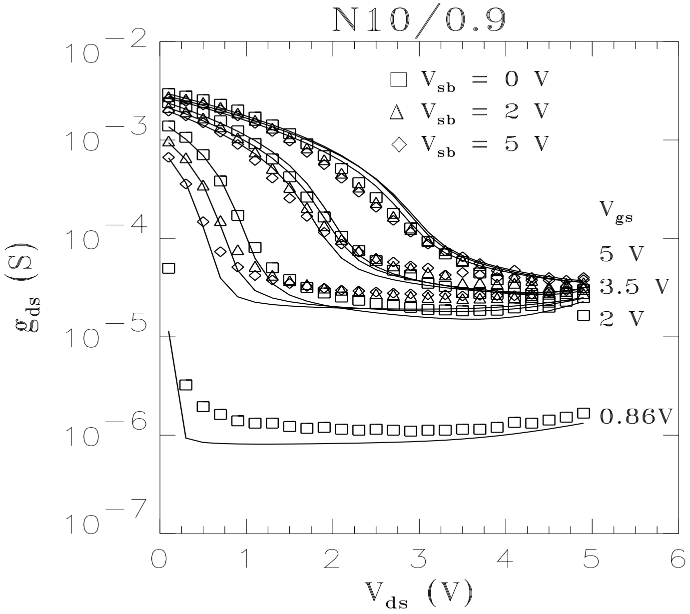


MOS MODEL 7
output conductance

- MOS MODEL 7 is only suited for digital applications

MOS MODEL 9

- has been developed for analog applications

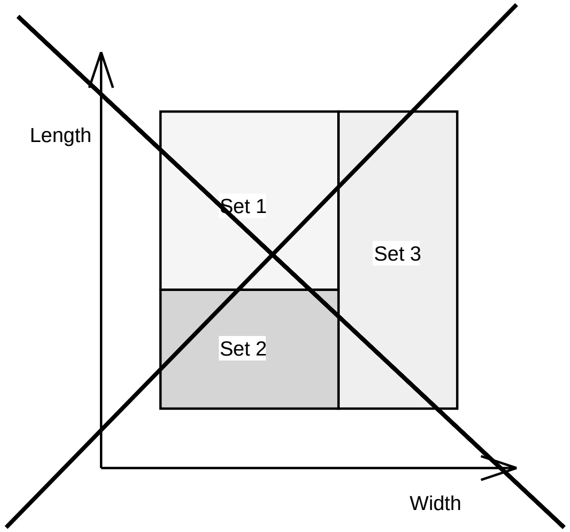


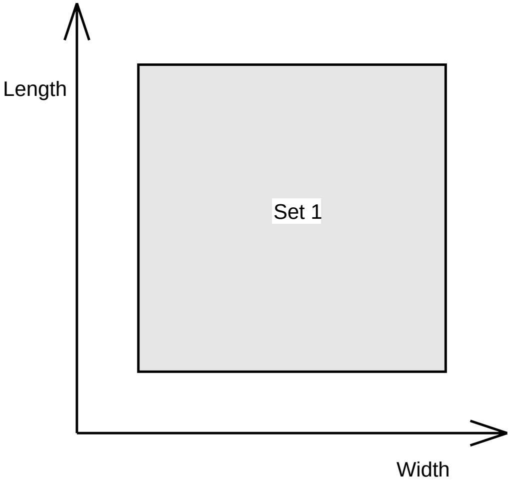
MOS MODEL 9
output conductance

- has continuous first and second order derivatives
- is suited for both digital and analog applications

MOS MODEL 9

- describes whole geometry range of process with **one** parameter set





- includes temperature scaling
- has no additional parameters for the charge model

MOS MODEL 9

- relation between physics and equations
  - parameters
  - scaling rules
- no exact, detailed derivations
- trends in specific parameters

linear region $V_{\mathsf{sb}} = 0V$

- drift current I_{ds} from continuity equation

$$I_{\mathsf{ds}} = -\mu \, W_{\mathsf{eff}} \, Q_{\mathsf{i}} \, \frac{\partial \phi}{\partial x}$$

- integration over x

$$\int_0^{L_{\mathsf{eff}}} I_{\mathsf{ds}} \, dx = L_{\mathsf{eff}} \, I_{\mathsf{ds}} = -\mu \, W_{\mathsf{eff}} \int_0^{V_{\mathsf{d}}} Q_{\mathsf{i}} \, d\phi$$

- intermediate result

$$I_{\mathsf{ds}} = -\mu \, \frac{W_{\mathsf{eff}}}{L_{\mathsf{eff}}} \int_0^{V_{\mathsf{d}}} Q_{\mathsf{i}} \, d\phi$$

- μ ⇒ carrier mobility; Q_{i} ⇒ inversion charge per unit surface

linear region $V_{\mathrm{sb}} = 0V$

- intermediate result

$$I_{\mathrm{ds}} \;=\; -\,\mu\,\frac{W_{\mathrm{eff}}}{L_{\mathrm{eff}}}\int_0^{V_{\mathrm{d}}} Q_{\mathrm{i}}\,d\phi$$

- inversion charge Q_{i} from Poisson equation

$$Q_{\mathrm{i}} \;\approx\; -\,C_{\mathrm{ox}}\,(V_{\mathrm{gs}} - V_{\mathrm{TO}} - \phi)$$

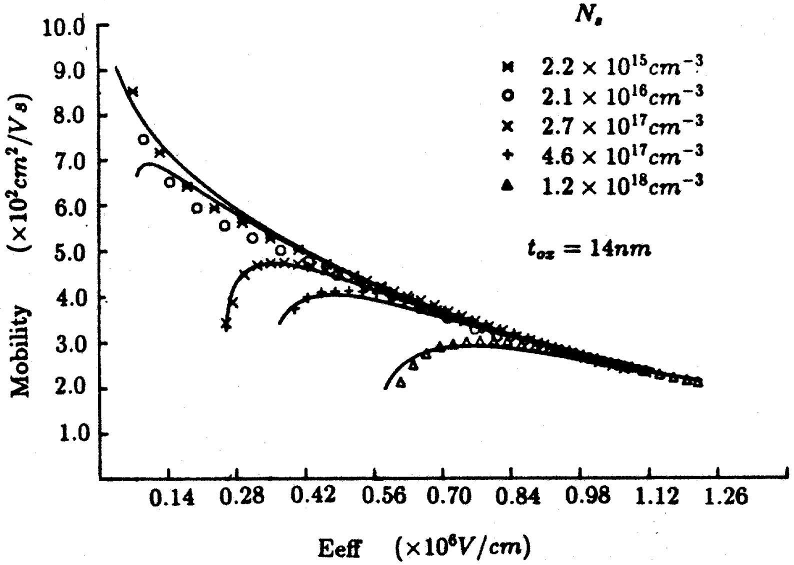
- substitution and integration

$$I_{\mathrm{ds}} \;=\; \mu\,C_{\mathrm{ox}}\,\frac{W_{\mathrm{eff}}}{L_{\mathrm{eff}}}\left[(V_{\mathrm{gs}} - V_{\mathrm{TO}})\,V_{\mathrm{d}} \;-\; \frac{1}{2}\,{V_{\mathrm{d}}}^2\right]$$

- C_{ox} ⇒ oxide capacitance per unit surface
  V_{TO} ⇒ MM9 threshold voltage

linear region $V_{\mathrm{sb}} = 0V$

- carrier mobility depends on electric field perpendicular to surface



carrier mobility vs. $E_\perp$

- mobility is a function of gate-source voltage

$$\mu = \frac{\mu_{\mathrm{O}}}{1 + \theta_1 \left(V_{\mathrm{gs}} - V_{\mathrm{TO}}\right)}$$

- $\theta_1 \Rightarrow$ MM9 mobility reduction parameter

linear region $V_{\mathrm{sb}} = 0V$

- MM9 expression for I_{ds}

$$I_{\mathrm{ds}} = \beta \, \frac{(V_{\mathrm{gs}} - V_{\mathrm{TO}})\,V_{\mathrm{ds}} - \left(\frac{1+\delta_1}{2}\right) V_{\mathrm{ds}}^2}{1 + \theta_1\,(V_{\mathrm{gs}} - V_{\mathrm{TO}})}$$

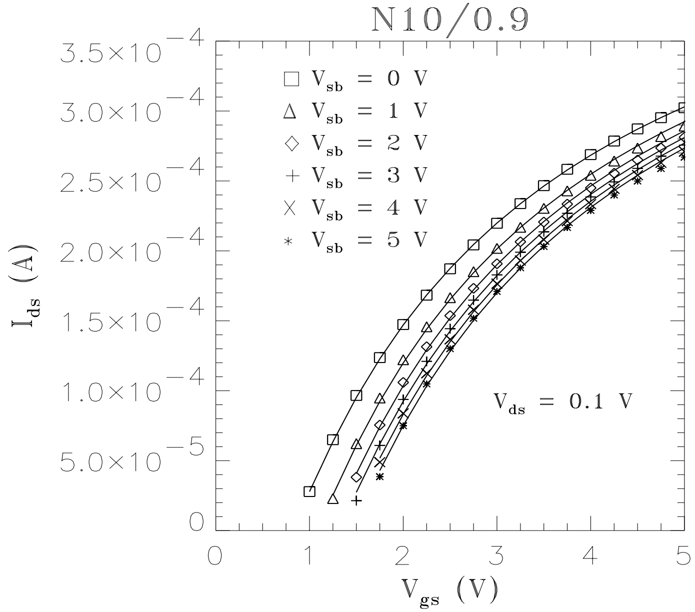
- β is the MM9 gain factor

$$\beta = \mu_{\mathrm{o}}\, C_{\mathrm{ox}} \frac{W_{\mathrm{eff}}}{L_{\mathrm{eff}}}$$

- charge in depletion layer increases with V_{ds}

$$\delta_1 \approx 0.3 \frac{k}{\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{b}}}}$$

linear region $V_{\mathrm{sb}} = 0V$



MOS MODEL 9
linear region

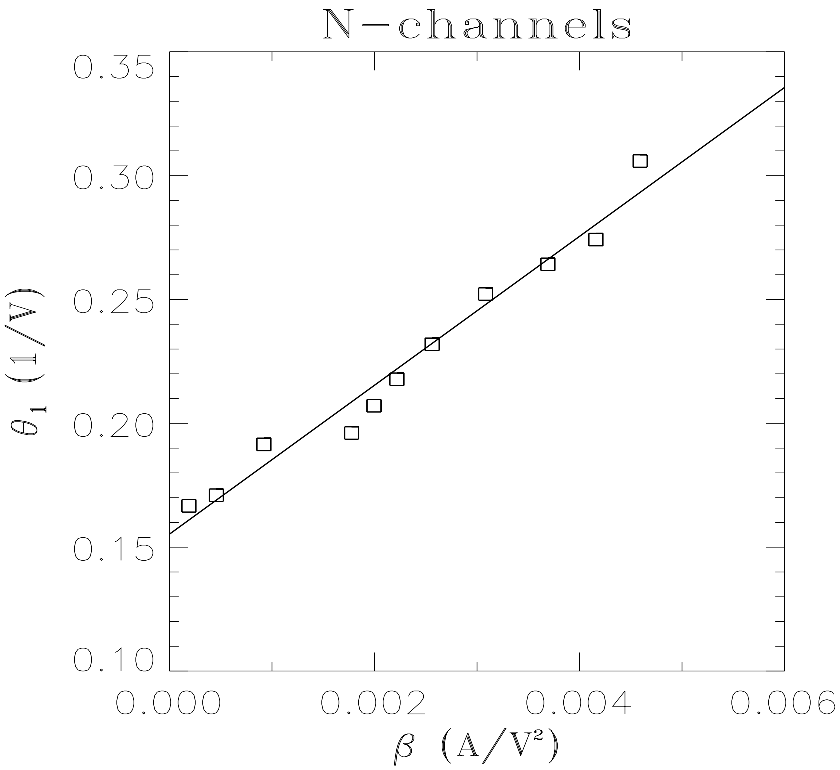
- MM9 expression for I_{ds}

$$I_{\mathrm{ds}} = \beta \, \frac{(V_{\mathrm{gs}} - V_{\mathrm{TO}})\, V_{\mathrm{ds}} - \left(\frac{1+\delta_1}{2}\right) V_{\mathrm{ds}}^2}{1 + \theta_1 \, (V_{\mathrm{gs}} - V_{\mathrm{TO}})}$$

- source and drain series resistances are included in θ_1

$$\theta_1 = \theta_{10} + \beta \, (R_{\mathrm{s}} + R_{\mathrm{d}})$$

linear region $V_{\mathrm{sb}} = 0V$

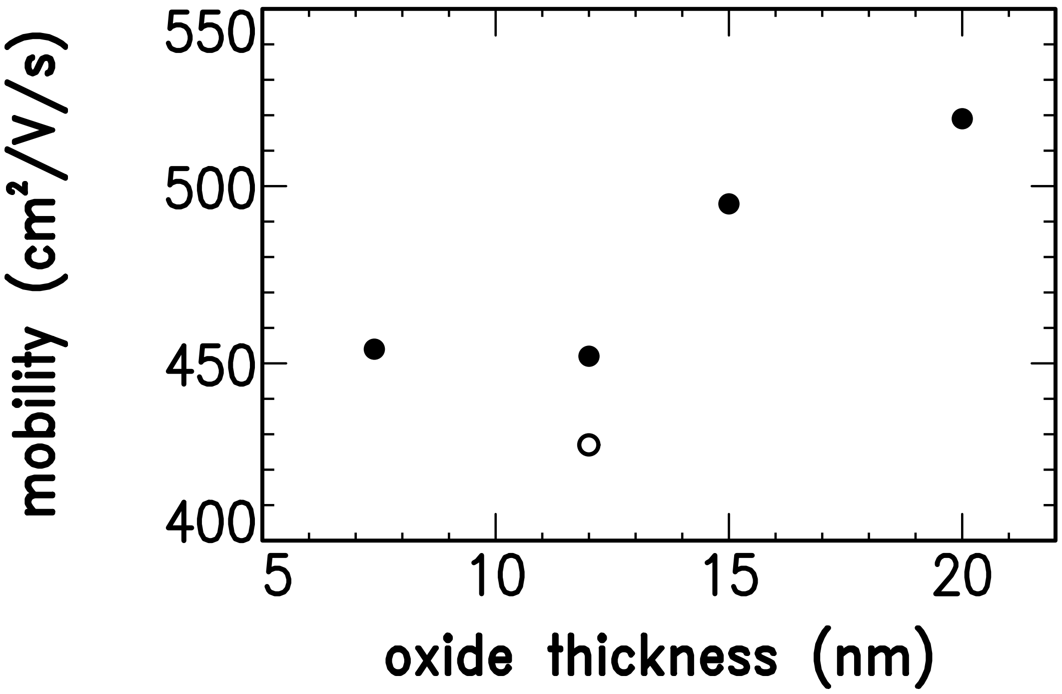


MOS MODEL 9

θ_1 vs. β

- source and drain series resistances are included in θ_1

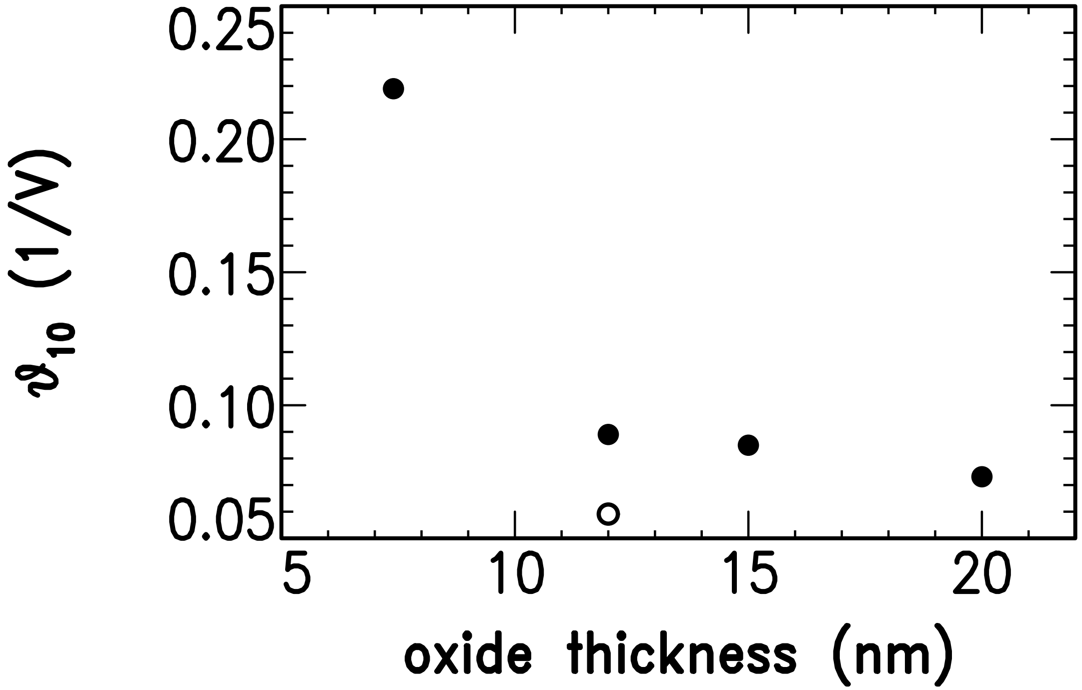
$$\theta_1 = \theta_{10} + \beta\,(R_{\mathrm{s}} + R_{\mathrm{d}})$$



- process dependence of the carrier mobility for N-channels

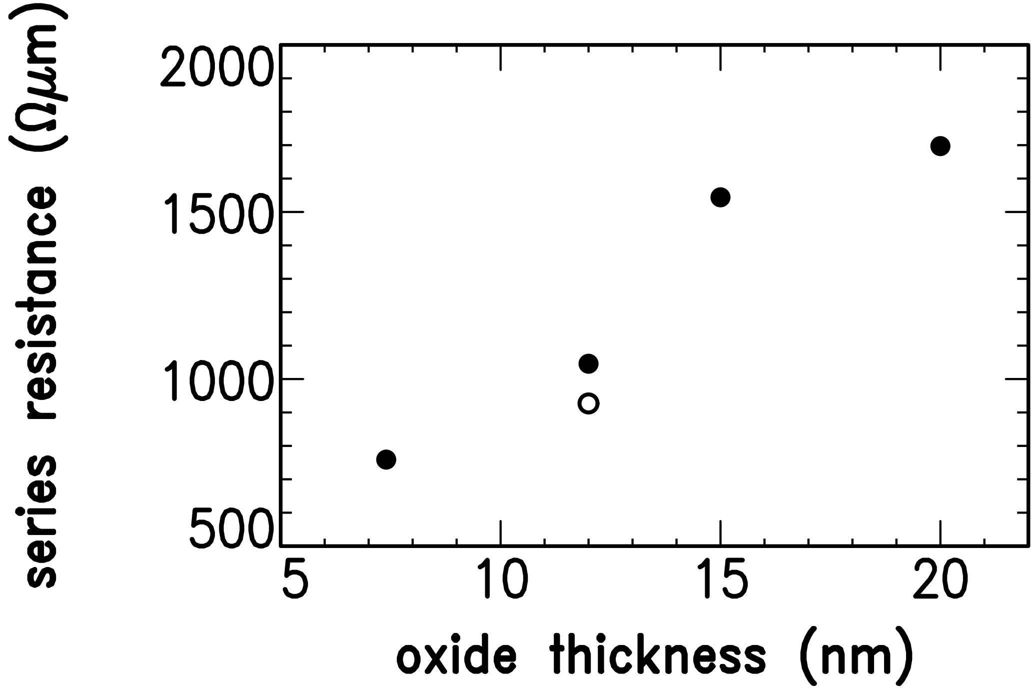
$$\mu_{\mathrm{o}} = \frac{\beta_{\square}}{C_{\mathrm{ox}}}$$

linear region $V_{\mathrm{sb}} = 0V$



- process dependence of the carrier mobility reduction parameter θ_1 for infinite long and wide N-channel transistor ($\Rightarrow \theta_{10}$)

linear region $V_{\mathrm{sb}} = 0V$



- process dependence of the series resistance for N-channels

$$R_{\mathrm{s}} + R_{\mathrm{d}} = \frac{\rho}{W_{\mathrm{eff}}} \quad \Rightarrow \quad \rho = \frac{S_{\mathrm{L};\theta_1,\mathrm{R}}}{\beta_{\square}}$$

linear region $V_{\mathrm{sb}} \neq 0V$

- inversion charge Q_{i} from Poisson equation

$$Q_{\mathrm{i}} \approx -C_{\mathrm{ox}} \left(V_{\mathrm{gs}} - V_{\mathrm{FB}} - \phi_{\mathrm{B}} - k\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{B}}} \right) = -C_{\mathrm{ox}} \left(V_{\mathrm{gs}} - V_{\mathrm{T1}} \right)$$

- threshold voltage V_{T1}

$$V_{\mathrm{T1}} = V_{\mathrm{FB}} + \phi_{\mathrm{B}} + k\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{B}}} = V_{\mathrm{T0}} + k\left(\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{B}}} - \sqrt{\phi_{\mathrm{B}}} \right)$$

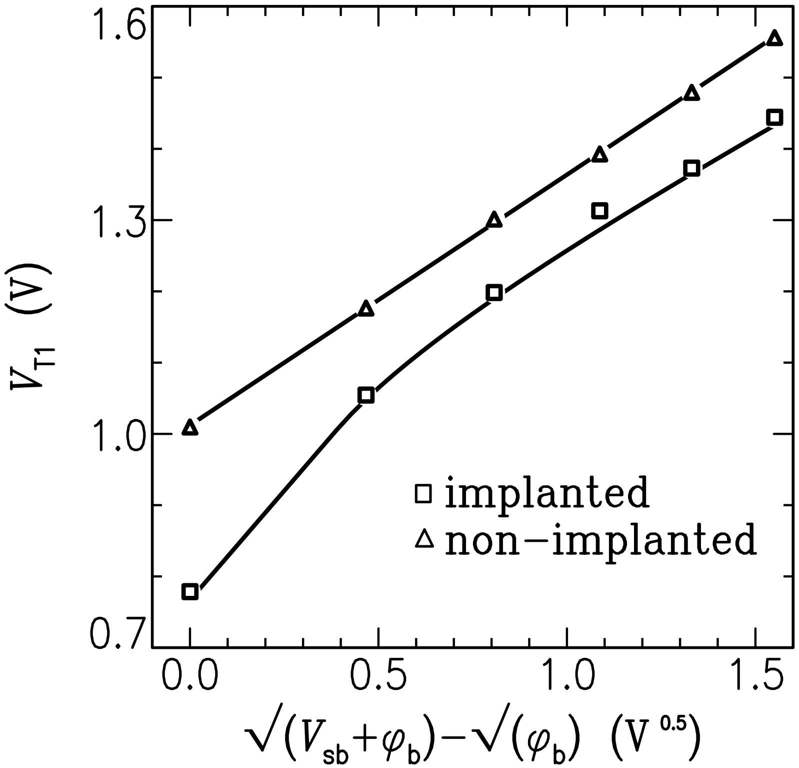
- MM9 threshold voltage parameter V_{T0}

$$V_{\mathrm{T0}} = V_{\mathrm{FB}} + \phi_{\mathrm{B}} + k\sqrt{\phi_{\mathrm{B}}}$$

- $V_{\mathrm{FB}} \Rightarrow$ flat band voltage
  $\phi_{\mathrm{B}} \Rightarrow$ surface potential in strong inversion $\approx 2\times$ bulk Fermi potential
- MM9 body-effect factor k depends on dopant concentration N

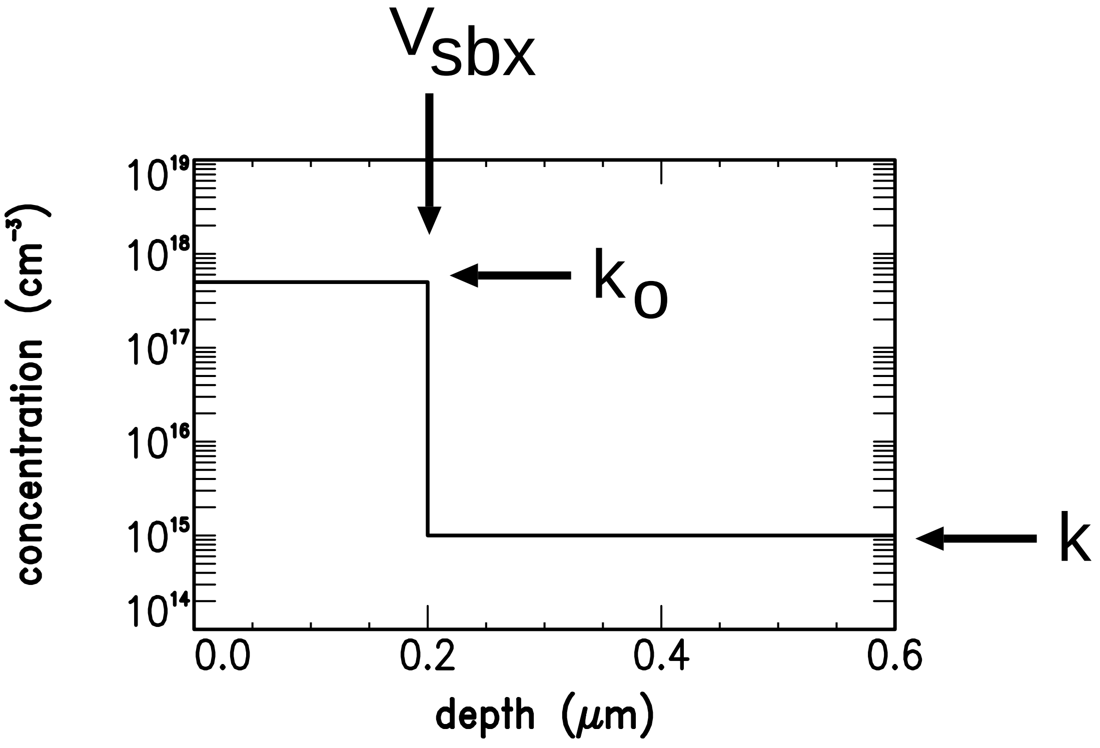
$$k = \frac{\sqrt{2\,\varepsilon_{\mathrm{s}}\,q\,N}}{C_{\mathrm{ox}}}$$

linear region $V_{\mathrm{sb}} \neq 0V$



- different behaviour for devices with a channel-implantation

linear region $V_{\mathrm{sb}} \neq 0V$



- with channel-implantation two MM9 body-effect factors: k_{o} and k
- MM9 parameter V_{sbx} depends on depth of channel-implantation

linear region $V_{sb} \neq 0V$

- back-bias affects electrical field perpendicular to surface

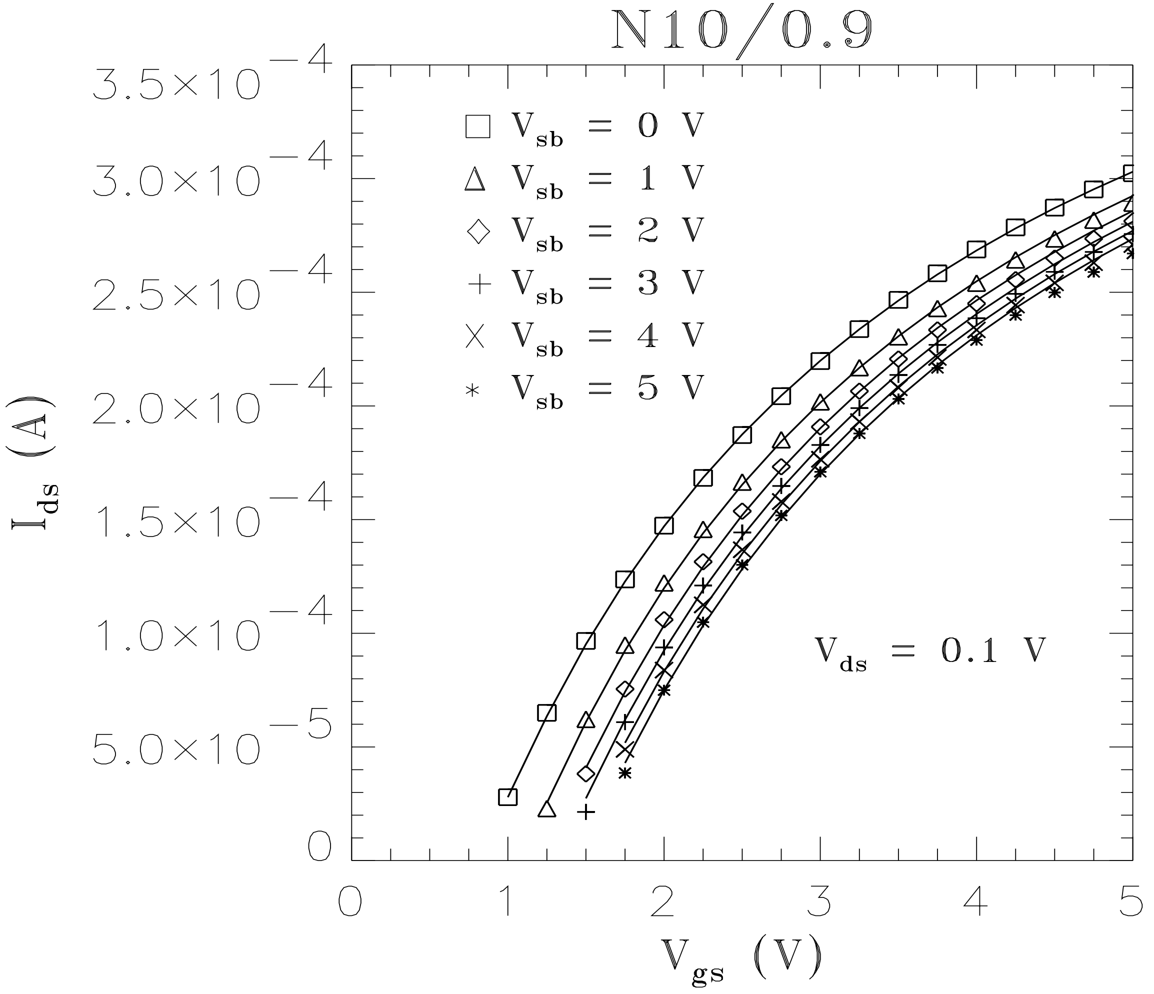
$$\mu = \frac{\mu_o}{1 + \theta_1 \left(V_{gs} - V_{T1}\right) + \theta_2 \left(\sqrt{V_{sb} + \phi_B} - \sqrt{\phi_B}\right)}$$

- $\theta_2 \Rightarrow$ second MM9 mobility reduction parameter

- MM9 expression for I_{ds}

$$I_{ds} = \beta \frac{\left(V_{gs} - V_{T1}\right) V_{ds} - \left(\frac{1+\delta_1}{2}\right) V_{ds}^2}{1 + \theta_1 \left(V_{gs} - V_{T1}\right) + \theta_2 \left(\sqrt{V_{sb} + \phi_B} - \sqrt{\phi_B}\right)}$$

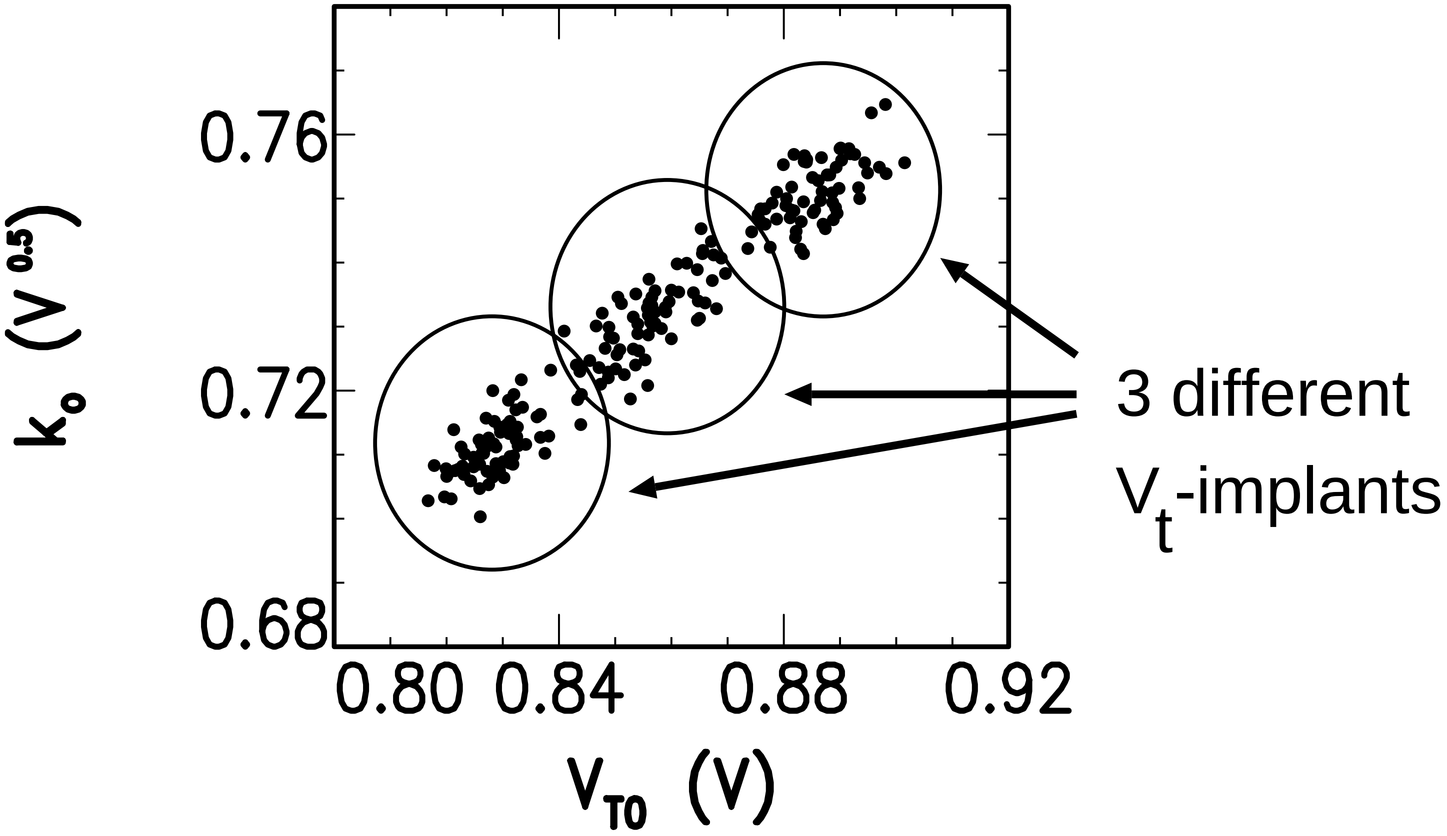
linear region $V_{sb} \neq 0V$



MOS MODEL 9

linear region

linear region $V_{\mathrm{sb}} \neq 0V$



- both V_{T0} and k_{o} depend on V_{T}- or channel-implantation

subthreshold region

- diffusion current I_{ds} from continuity equation

$$I_{\mathsf{ds}} = -D\,W_{\mathsf{eff}}\,\frac{\partial Q_{\mathsf{i}}}{\partial x} = -\phi_{\mathsf{T}}\,\mu\,W_{\mathsf{eff}}\,\frac{\partial Q_{\mathsf{i}}}{\partial x}$$

- integration over x

$$\int_0^{L_{\mathsf{eff}}} I_{\mathsf{ds}}\,dx = L_{\mathsf{eff}}\,I_{\mathsf{ds}} = -\phi_{\mathsf{T}}\,\mu\,W_{\mathsf{eff}}\int_0^{L_{\mathsf{eff}}}\frac{\partial Q_{\mathsf{i}}}{\partial x}\,dx$$

$$L_{\mathsf{eff}}\,I_{\mathsf{ds}} = -\phi_{\mathsf{T}}\,\mu\,W_{\mathsf{eff}}\left\{Q_{\mathsf{i\,D}} - Q_{\mathsf{i\,S}}\right\}$$

- intermediate result

$$I_{\mathsf{ds}} = \phi_{\mathsf{T}}\,\mu\,\frac{W_{\mathsf{eff}}}{L_{\mathsf{eff}}}\left\{Q_{\mathsf{i\,S}} - Q_{\mathsf{i\,D}}\right\}$$

- $\mu \Rightarrow$ carrier mobility; $Q_{\mathsf{i}} \Rightarrow$ inversion charge per unit surface; $\phi_{\mathsf{T}} \Rightarrow$ thermal voltage

subthreshold region

- intermediate result

$$I_{\mathrm{ds}} = \phi_{\mathrm{T}}\, \mu\, \frac{W_{\mathrm{eff}}}{L_{\mathrm{eff}}} \left\{ Q_{\mathrm{i\,S}} - Q_{\mathrm{i\,D}} \right\}$$

- inversion charge Q_{i} from Poisson equation

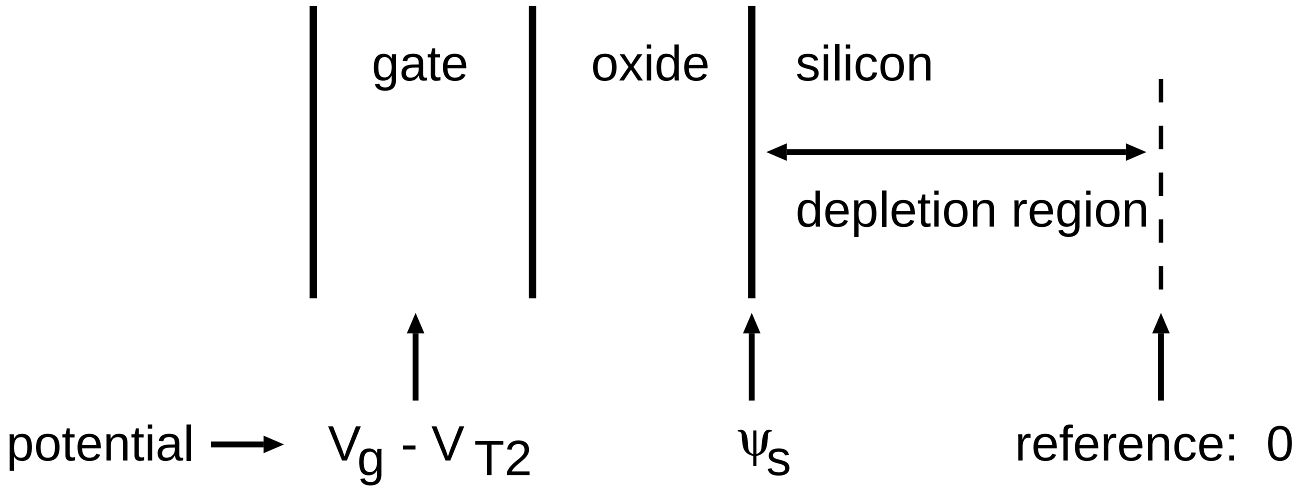
$$Q_{\mathrm{i}} \approx Q_{\mathrm{i\,0}} \exp\left(\frac{\psi_{\mathrm{s}}}{\phi_{\mathrm{T}}}\right) \quad \Rightarrow \quad Q_{\mathrm{i\,D}} = Q_{\mathrm{i\,S}} \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right)$$

- substitution

$$I_{\mathrm{ds}} = \phi_{\mathrm{T}}\, \mu\, \frac{W_{\mathrm{eff}}}{L_{\mathrm{eff}}}\, Q_{\mathrm{i\,0}} \exp\left(\frac{\psi_{\mathrm{s\,S}}}{\phi_{\mathrm{T}}}\right) \left\{ 1 - \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right) \right\}$$

- $\psi_{\mathrm{s}} \Rightarrow$ surface potential; $\psi_{\mathrm{s\,S}}$ surface potential at source-side

subthreshold region



- dependence of surface potential on gate voltage

$$\psi_{sS} = \frac{C_{ox}\,(V_{gs} - V_{T2})}{C_{ox} + C_{depl}} = \frac{V_{gt2}}{m}$$

- the subthreshold slope m is given by

$$m = 1 + \frac{C_{depl}}{C_{ox}}$$

subthreshold region

- from "theory"

$$I_{\mathrm{ds}} = \phi_{\mathrm{T}}\,\mu\,\frac{W_{\mathrm{eff}}}{L_{\mathrm{eff}}}\,Q_{\mathrm{i}\,0}\,\exp\left(\frac{V_{\mathrm{gt2}}}{m\,\phi_{\mathrm{T}}}\right)\left\{1 - \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right)\right\}$$

- MM9 expression for I_{ds}

$$I_{\mathrm{ds}} = \beta\,G_{3}\,\frac{V_{\mathrm{gt3}}\,V_{\mathrm{ds1}} - \left(\frac{1+\delta_1}{2}\right)\,V_{\mathrm{ds1}}^{2}}{\left\{1 + \theta_1\,V_{\mathrm{gt1}} + \theta_2\,\left(\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{B}}} - \sqrt{\phi_{\mathrm{B}}}\right)\right\}\,(1 + \theta_3\,V_{\mathrm{ds1}})}$$

subthreshold region

- MM9 expression for I_{ds}

$$I_{\mathrm{ds}} = \beta\, G_3 \; \frac{V_{\mathrm{gt3}}\, V_{\mathrm{ds1}} \;-\; \left(\frac{1+\delta_1}{2}\right) V_{\mathrm{ds1}}^2}{\left\{1 + \theta_1\, V_{\mathrm{gt1}} + \theta_2 \left(\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{B}}} - \sqrt{\phi_{\mathrm{B}}}\right)\right\} (1 + \theta_3\, V_{\mathrm{ds1}})}$$

- herein $V_{\mathrm{gt1}} = \max\{0,\, (V_{\mathrm{gs}} - V_{\mathrm{T1}})\}$

$$G_1 = \exp\left(\frac{V_{\mathrm{gt2}}}{2\, m\, \phi_{\mathrm{T}}}\right)$$

$$V_{\mathrm{gt3}} = 2\, m\, \phi_{\mathrm{T}} \,\ln(1 + G_1)$$

$$V_{\mathrm{gt2}} \ll -m\, \phi_{\mathrm{T}} \Rightarrow V_{\mathrm{gt3}} = 2\, m\, \phi_{\mathrm{T}} \,\exp\left(\frac{V_{\mathrm{gt2}}}{2\, m\, \phi_{\mathrm{T}}}\right)$$

$$V_{\mathrm{gt2}} \gg -m\, \phi_{\mathrm{T}} \Rightarrow V_{\mathrm{gt3}} = V_{\mathrm{gt2}}$$

$$V_{\mathrm{ds1}} = \frac{1}{1+\delta_1} V_{\mathrm{gt3}}$$

$$G_3 = \frac{\zeta_1 \left\{1 - \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right)\right\} + G_1}{\frac{1}{\zeta_1} + G_1} \quad \Rightarrow \quad \zeta_1^2 \left\{1 - \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right)\right\}$$

subthreshold region

- MM9 expression for I_{ds}

$$I_{\mathrm{ds}} = \zeta_1^2\,\beta\left\{1 - \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right)\right\}\;\frac{\frac{1}{2}\,\frac{1}{1+\delta_1}\,V_{\mathrm{gt3}}^2}{\{1+\theta_2\,(\sqrt{V_{\mathrm{sb}}+\phi_{\mathrm{B}}}-\sqrt{\phi_{\mathrm{B}}})\}}$$

$$I_{\mathrm{ds}} = \zeta_1^2\,\beta\left\{1 - \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right)\right\}\;\frac{\frac{2\,m^2\,\phi_{\mathrm{T}}^2}{1+\delta_1}\,\exp\left(\frac{V_{\mathrm{gt2}}}{m\,\phi_{\mathrm{T}}}\right)}{\{1+\theta_2\,(\sqrt{V_{\mathrm{sb}}+\phi_{\mathrm{B}}}-\sqrt{\phi_{\mathrm{B}}})\}}$$

- from “theory”

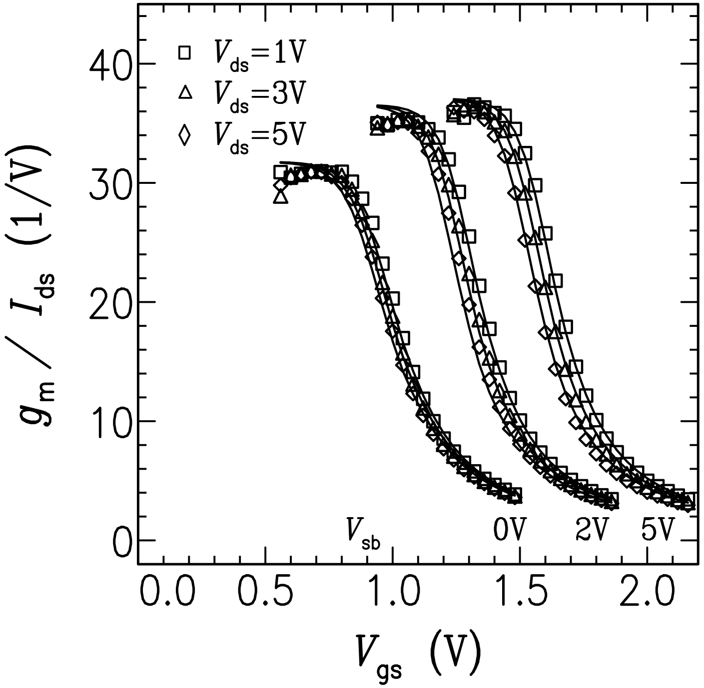
$$I_{\mathrm{ds}} = \phi_{\mathrm{T}}\,\mu\,\frac{W_{\mathrm{eff}}}{L_{\mathrm{eff}}}\,Q_{\mathrm{i}\,0}\,\exp\left(\frac{V_{\mathrm{gt2}}}{m\,\phi_{\mathrm{T}}}\right)\left\{1 - \exp\left(-\frac{V_{\mathrm{ds}}}{\phi_{\mathrm{T}}}\right)\right\}$$

- ζ_1 is the “weak-inversion correction” parameter of MM9

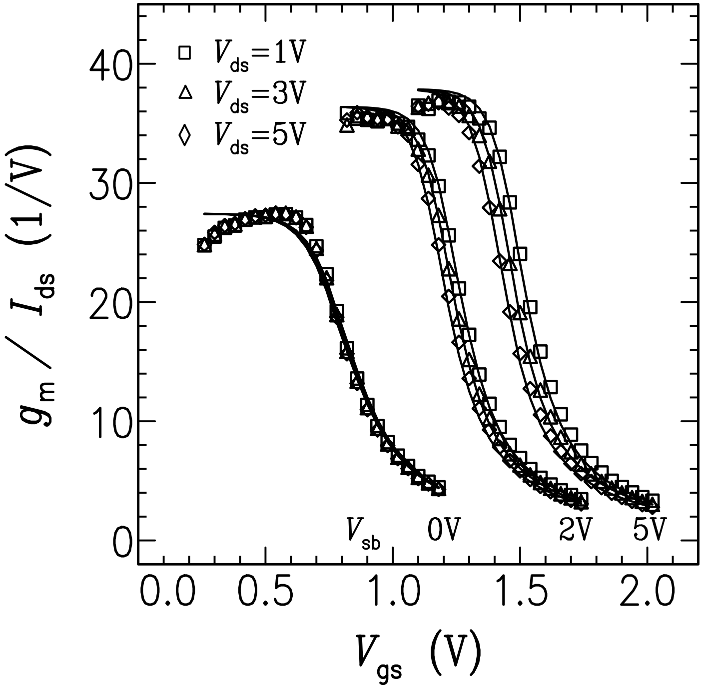
subthreshold region

- the subthreshold slope m can be determined from

$$m = \left(\phi_{\mathrm{T}} \frac{\mathrm{d} \ln I_{\mathrm{ds}}}{\mathrm{d} V_{\mathrm{gs}}}\right)^{-1} = \left(\phi_{\mathrm{T}} \frac{g_{\mathrm{m}}}{I_{\mathrm{ds}}}\right)^{-1}$$



non-implanted substrate

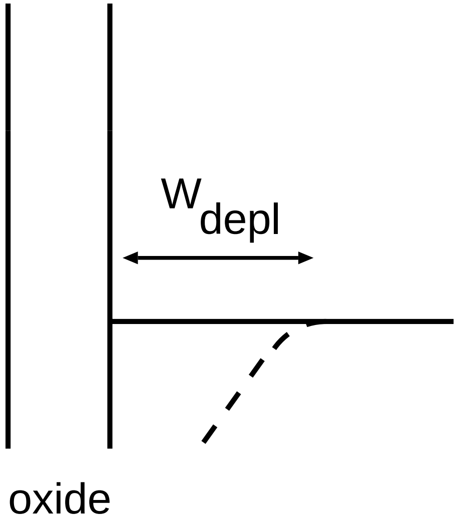


implanted substrate

subthreshold region

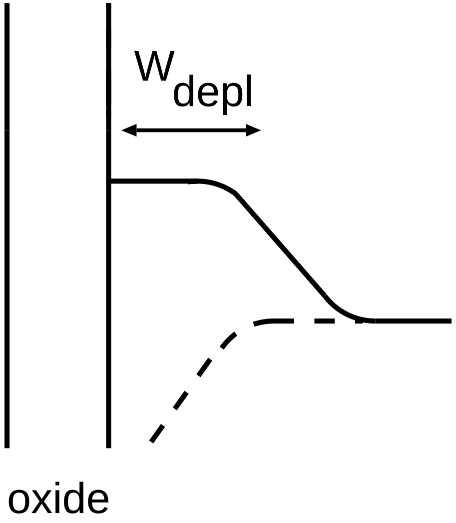
- the subthreshold slope m is given by

$$m = 1 + \frac{C_{\text{depl}}}{C_{\text{ox}}}$$



non-implanted substrate

$$C_{\text{depl}} \propto \frac{1}{\sqrt{V_{\text{sb}}+\phi_{\text{B}}}}$$



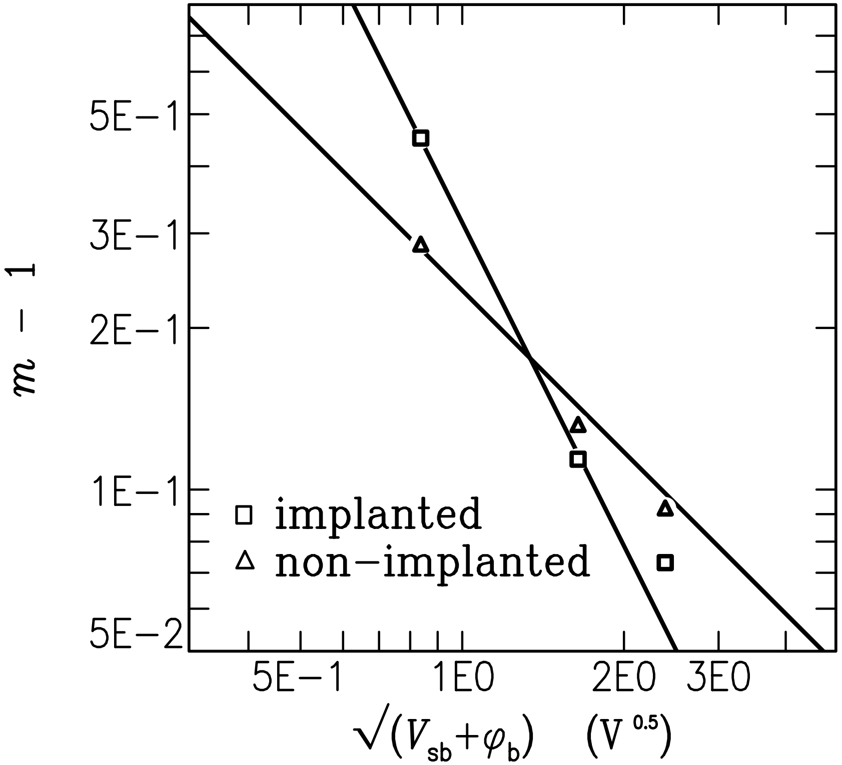
implanted substrate

$$C_{\text{depl}} \propto \frac{1}{V_{\text{sb}}+\phi_{\text{B}}}$$

- in MM9 the subthreshold slope m is modelled by

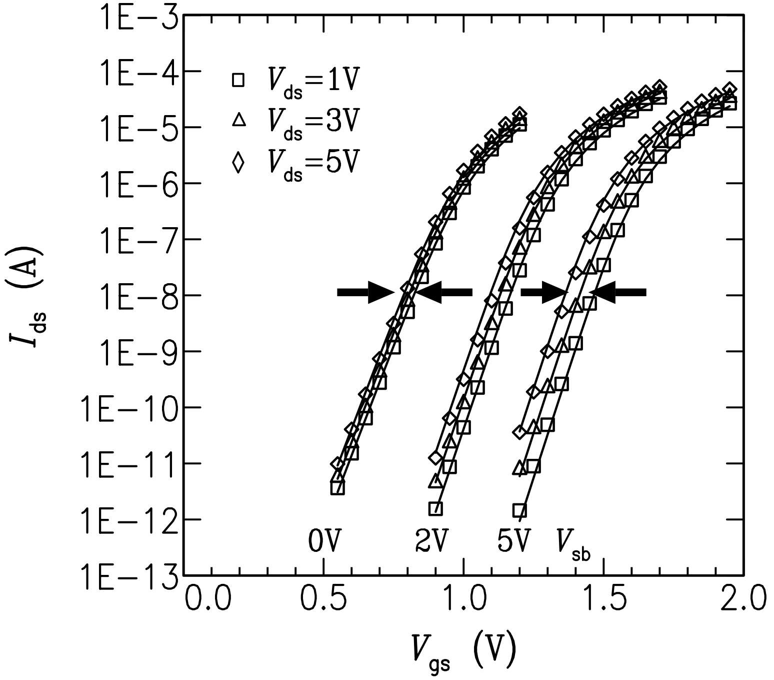
$$m = 1 + m_{\mathrm{o}} \left(\frac{\sqrt{\phi_{\mathrm{b}}}}{\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{b}}}} \right)^{\eta_{\mathrm{m}}}$$

- non-impl. substrate ⇒ $\eta_{\mathrm{m}} = 1$
- implanted substrate ⇒ $\eta_{\mathrm{m}} = 2$
- m_{o} ⇒ MM9 subthreshold-slope parameter

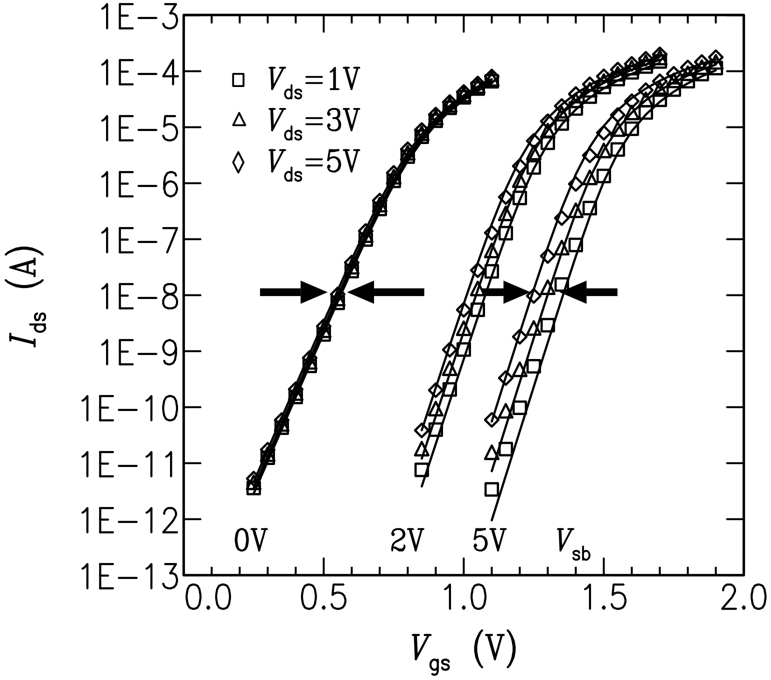


subthreshold region

- threshold shift increasing with V_{ds}



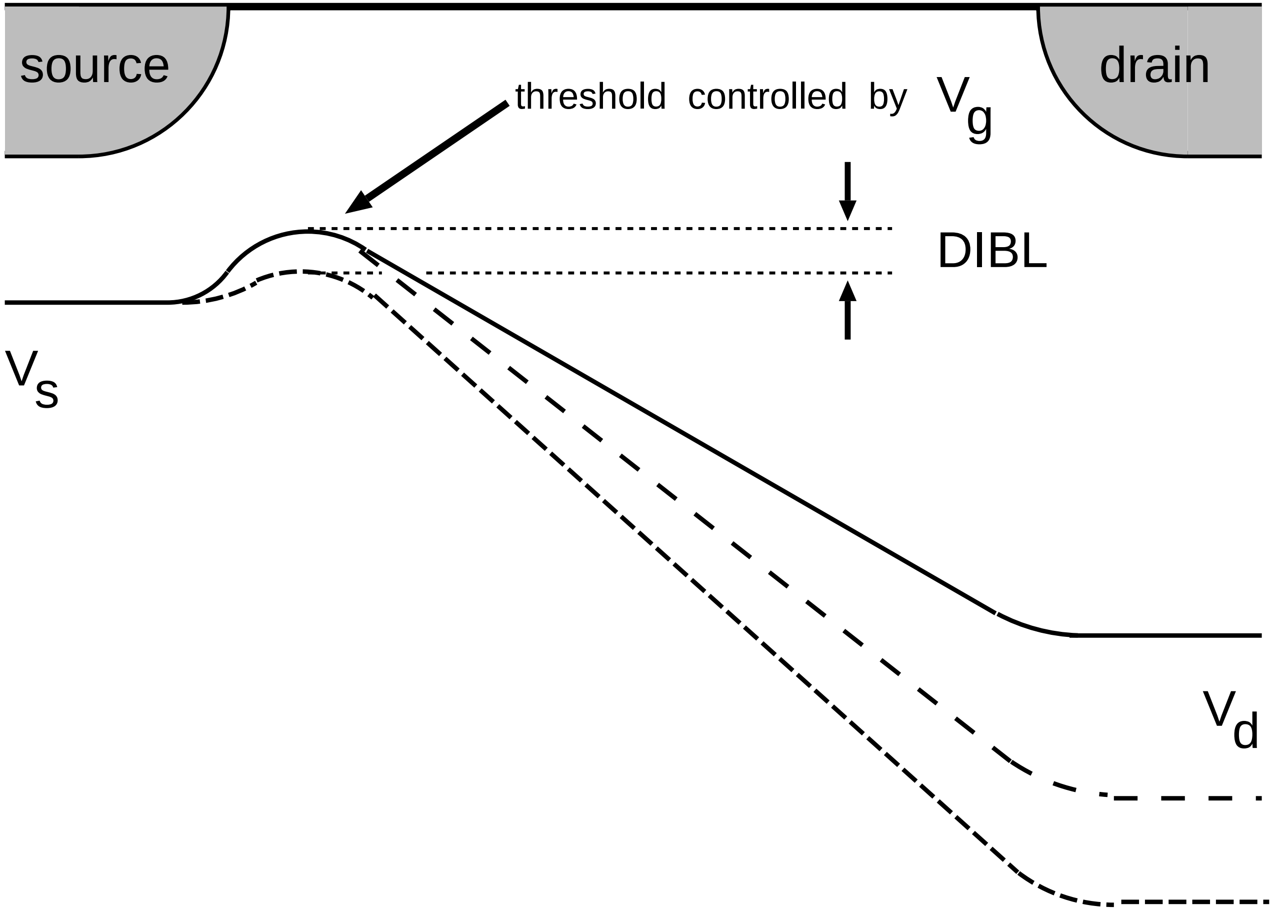
non-implanted substrate



implanted substrate

subthreshold region

- threshold shift increasing with V_{ds}



subthreshold region

- theoretical drain-induced barrier-lowering for non-implanted substrate

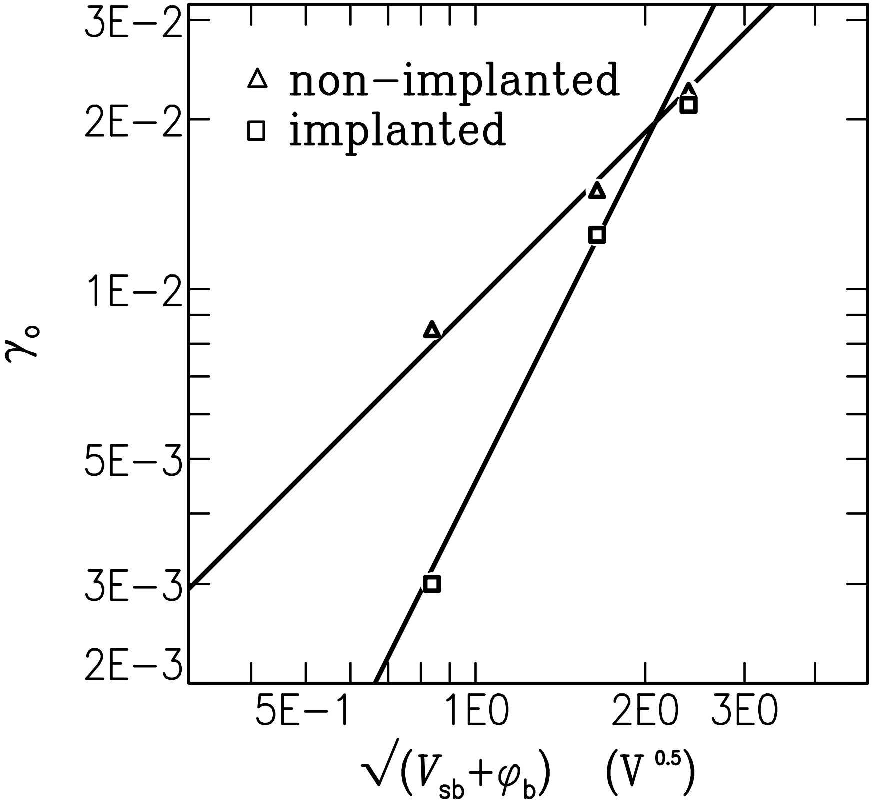
$$\Delta V_{\mathrm{T,\,DIBL}} \propto -\frac{\sqrt{V_{\mathrm{sb}}+\phi_{\mathrm{b}}}}{L_{\mathrm{eff}}^{2}}\, V_{\mathrm{ds}}$$

- DIBL in MOS MODEL 9

$$\Delta V_{\mathrm{T,\,DIBL}} = -\gamma_{\mathrm{o}}\, V_{\mathrm{ds}}$$

$$\gamma_{\mathrm{o}} = \gamma_{\mathrm{oo}} \left(\frac{\sqrt{V_{\mathrm{sb}}+\phi_{\mathrm{b}}}}{\sqrt{\phi_{\mathrm{b}}}} \right)^{\eta_\gamma}$$

- non-impl. substrate ⇒ $\eta_\gamma = 1$
- implanted substrate ⇒ $\eta_\gamma = 2$
- γ_{oo} ⇒ MM9 DIBL parameter



subthreshold region

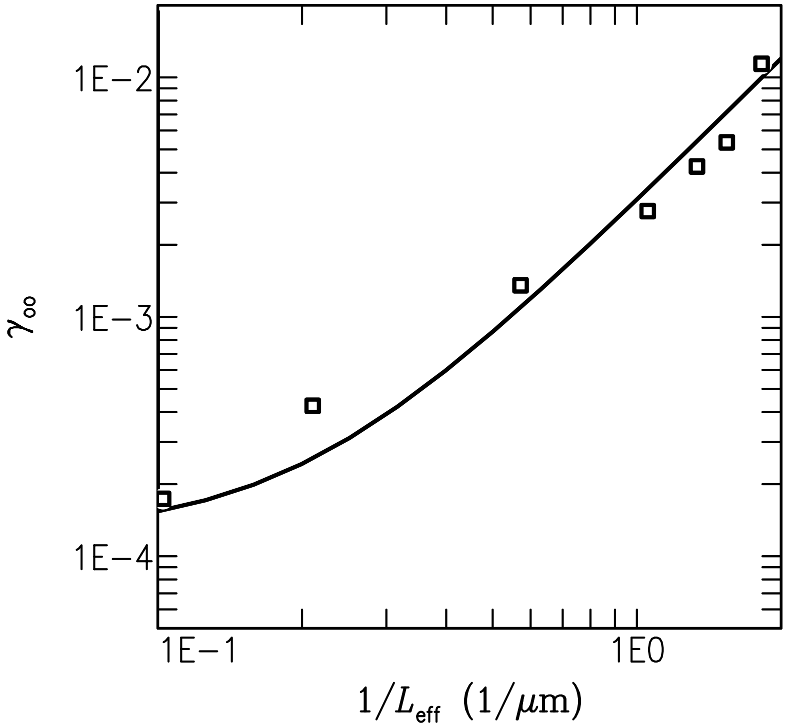
- theoretical drain-induced barrier-lowering for non-implanted substrate

$$\Delta V_{\mathrm{T,\,DIBL}} \propto - \frac{\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{b}}}}{L_{\mathrm{eff}}^{2}} V_{\mathrm{ds}}$$

- scaling of DIBL in MM9

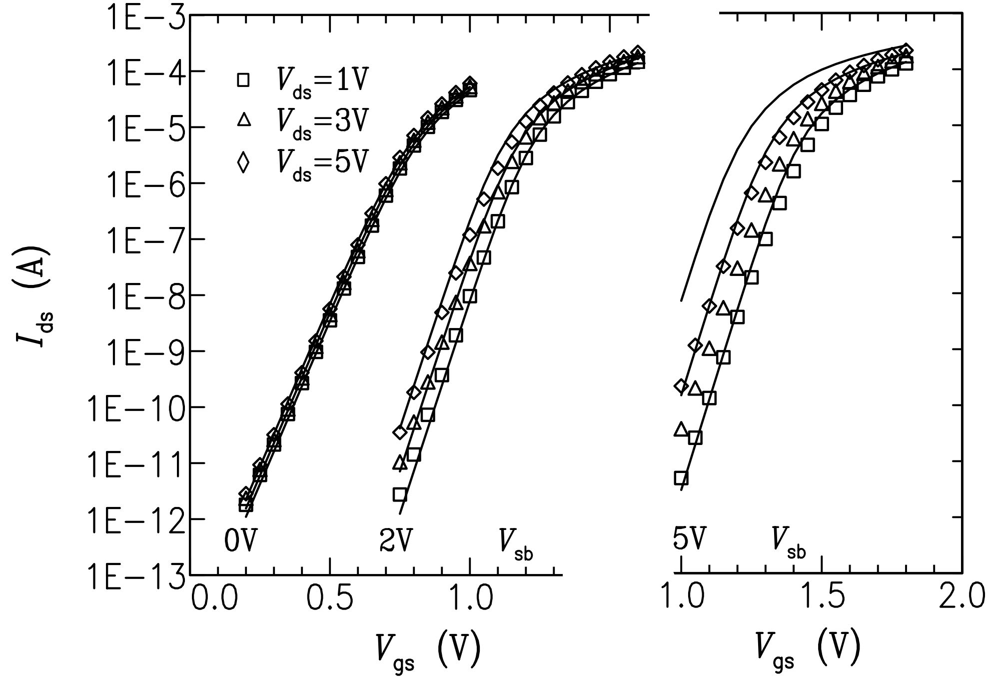
$$\gamma_{\mathrm{oo}} = \gamma_{\mathrm{ooR}} + \left(\frac{1}{L_{\mathrm{eff}}^{2}} - \frac{1}{L_{\mathrm{effR}}^{2}} \right) S_{\mathrm{L};\,\gamma_{\mathrm{oo}}}$$

- example for implanted substrate



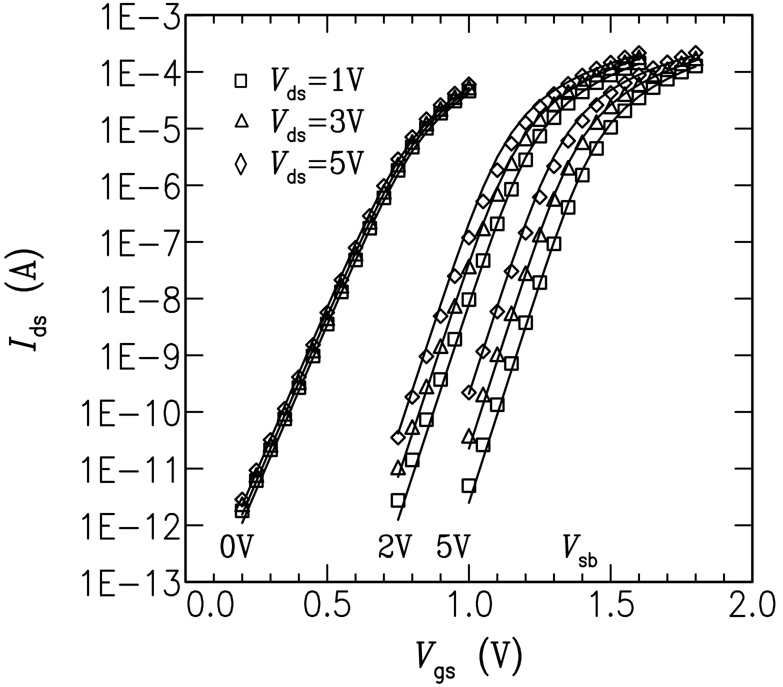
subthreshold region

- very small channel-lengths: source & drain depletion regions almost touch



without upper limit

for V_{sb} dependence



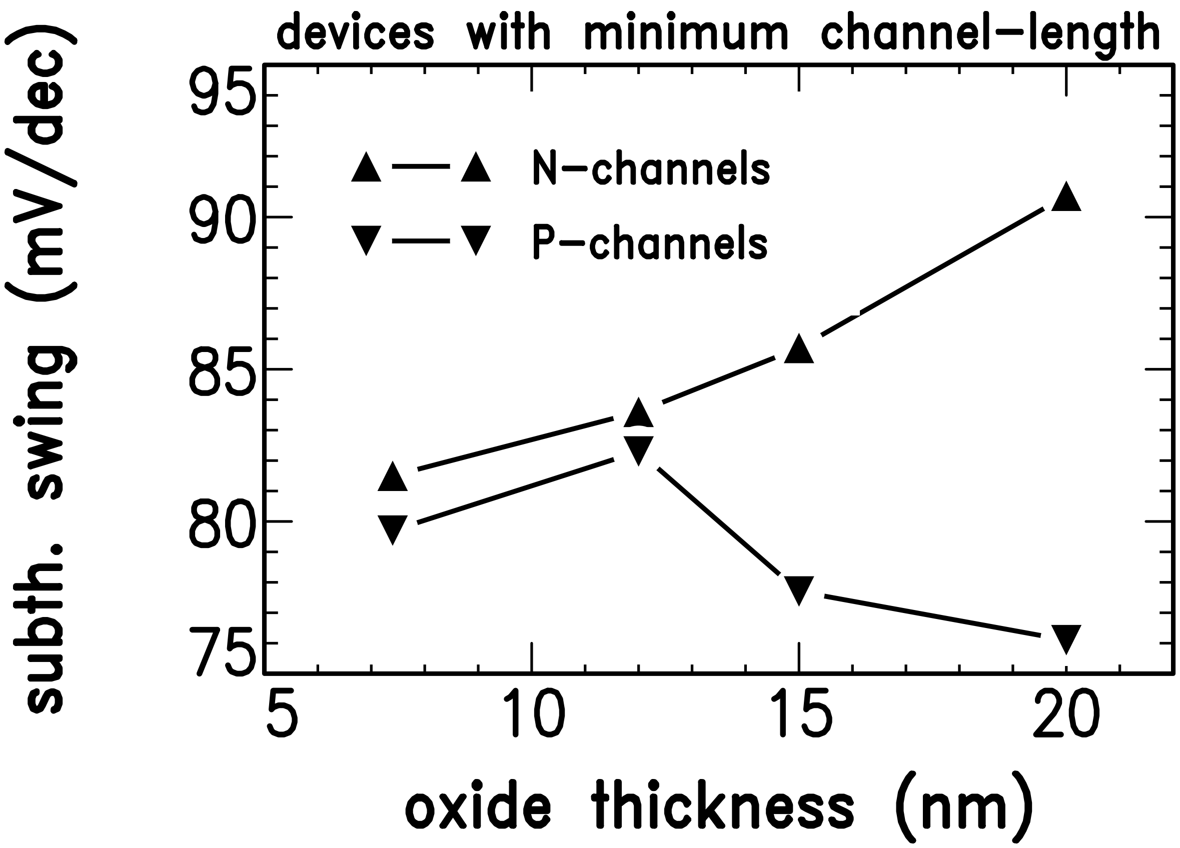
with upper limit

for V_{sb} dependence

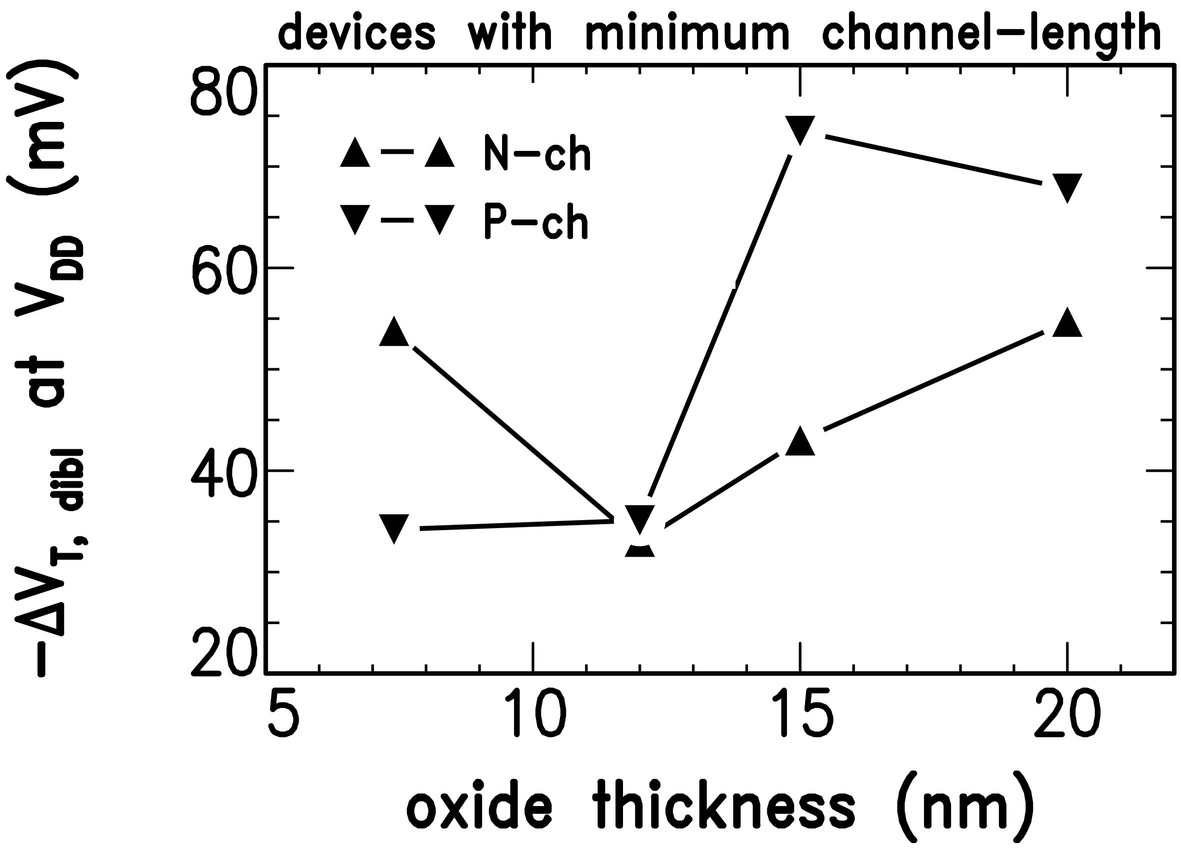
subthreshold description in MOS MODEL 9

- $V_{\mathrm{sb}} \leq V_{\mathrm{SBT}}$
  - $\gamma_{\mathrm{o}} = \gamma_{\mathrm{oo}} \left(\frac{\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{b}}}}{\sqrt{\phi_{\mathrm{b}}}} \right)^{\eta\gamma}$
  - $m = 1 + m_{\mathrm{o}} \left(\frac{\sqrt{\phi_{\mathrm{b}}}}{\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{b}}}} \right)^{\eta \mathrm{m}}$
- $V_{\mathrm{sb}} > V_{\mathrm{SBT}}$
  - $\gamma_{\mathrm{o}} = \gamma_{\mathrm{oo}} \left(\frac{\sqrt{V_{\mathrm{SBT}} + \phi_{\mathrm{b}}}}{\sqrt{\phi_{\mathrm{b}}}} \right)^{\eta\gamma}$
  - $m = 1 + m_{\mathrm{o}} \left(\frac{\sqrt{\phi_{\mathrm{b}}}}{\sqrt{V_{\mathrm{SBT}} + \phi_{\mathrm{b}}}} \right)^{\eta \mathrm{m}}$
- non-impl. substrate $\Rightarrow \eta_{\gamma} = \eta_{\mathrm{m}} = 1$
  implanted substrate $\Rightarrow \eta_{\gamma} = \eta_{\mathrm{m}} = 2$
- MM9 parameter V_{SBT} important only for smallest channel lengths

subthreshold region



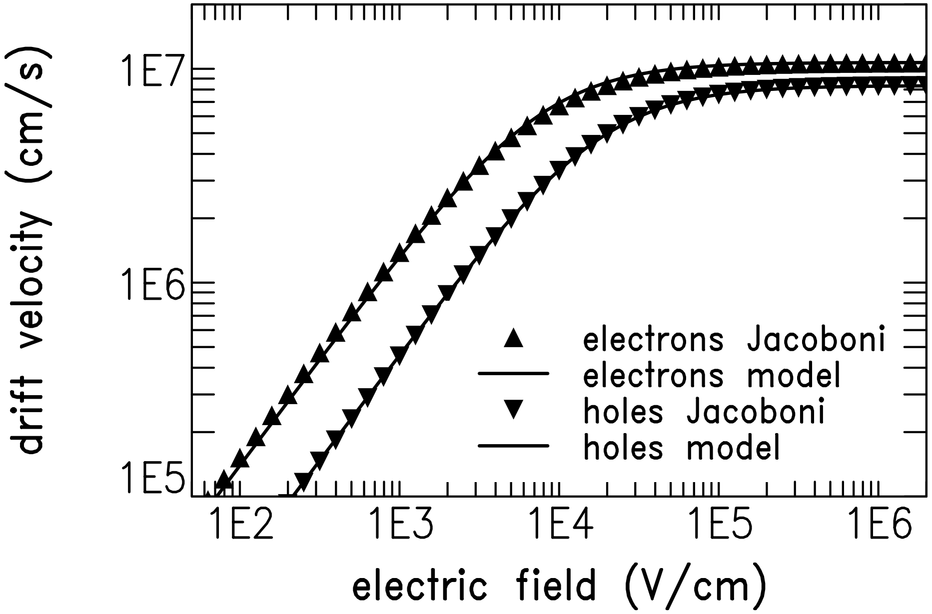
- subthreshold swing $S = (1 + m_{\mathrm{o}})\ \phi_{\mathrm{T}} \ln 10$



- drain-induced barrier-lowering at V_{dd}: $\Delta V_{T,\,DIBL} = -\gamma_{oo}\, V_{dd}$

saturation region

- at high electrical fields the drift velocity saturates



drift velocity vs. $E_{\|}$

- relation between drift velocity v, mobility μ and electric field $E_{\|}$

$$v = \mu\left(E_{\|}\right) E_{\|} = \frac{\mu E_{\|}}{1 + \frac{E_{\|}}{E_{\text{sat}}}} \quad \Rightarrow \quad \mu\left(E_{\|}\right) = \frac{\mu}{1 + \frac{E_{\|}}{E_{\text{sat}}}}$$

saturation region

- carrier mobility depends on drain-source voltage

$$\mu\left(E_{\parallel}\right) = \frac{\mu}{1 + \frac{E_{\parallel}}{E_{\mathrm{sat}}}} = \frac{\mu}{1 + \frac{V_{\mathrm{ds}}}{L_{\mathrm{eff}}\, E_{\mathrm{sat}}}} = \frac{\mu}{1 + \theta_3 V_{\mathrm{ds}}}$$

- θ_3 is the third MM9 mobility reduction parameter

$$\theta_3 = \frac{1}{L_{\mathrm{eff}}\, E_{\mathrm{sat}}}$$

- drift current I_{ds}

$$I_{\mathrm{ds}} = -\mu\, W_{\mathrm{eff}}\, Q_{\mathrm{i}} \frac{\partial \phi}{\partial x} = -\,v\, W_{\mathrm{eff}}\, Q_{\mathrm{i}}$$

saturates due to pinch-off and velocity saturation

saturation region

- expression for $V_{\text{ds, sat}}$ from expression for I_{ds}

$$I_{\text{ds}} = \mu C_{\text{ox}} \frac{W_{\text{eff}}}{L_{\text{eff}}} \left[(V_{\text{gs}} - V_{\text{TO}}) V_{\text{d}} - \frac{1}{2} V_{\text{d}}^2 \right] \Rightarrow V_{\text{ds, sat}} = V_{\text{gs}} - V_{\text{TO}}$$

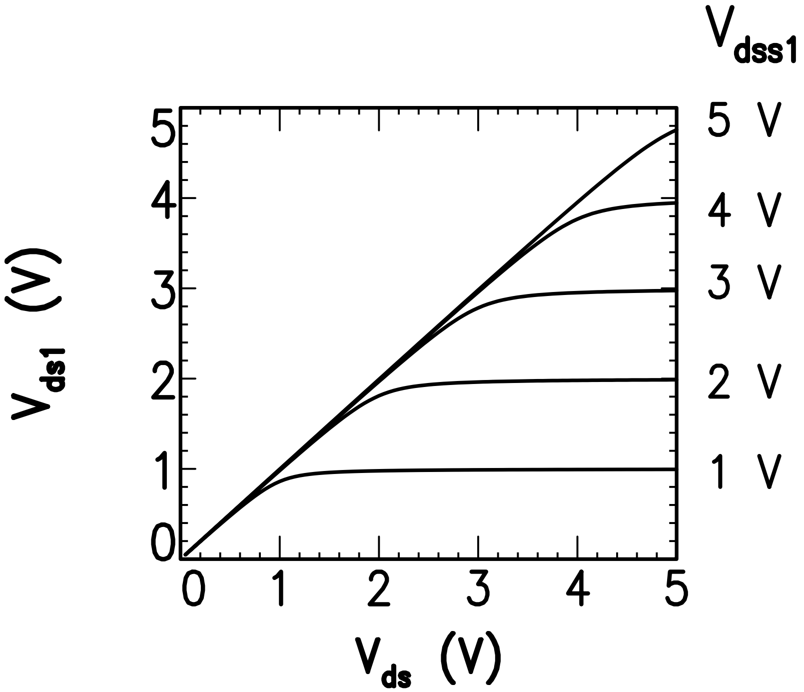
- MM9 expression for $V_{\text{ds, sat}}$

$$V_{\text{dss1}} = \frac{V_{\text{gt3}}}{1+\delta_1} \frac{2}{1+\sqrt{1+\frac{2\,\theta_3\,V_{\text{gt3}}}{1+\delta_1}}}$$

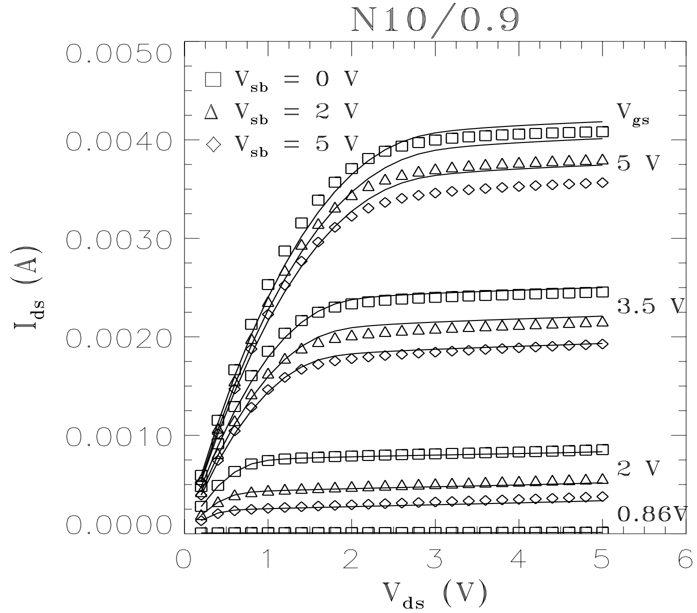
- effective source-gate voltage V_{gt3}

$$V_{\text{gt3}} = V_{\text{gs}} - V_{\text{T2}} \approx V_{\text{gs}} - V_{\text{T1}}$$

- V_{ds1} changes smoothly from V_{ds} to V_{dss1}



saturation region



MOS MODEL 9
saturation region

- MM9 expression for I_{ds}

$$I_{\mathrm{ds}} = \beta \frac{V_{\mathrm{gt3}} V_{\mathrm{ds1}} - \left(\frac{1+\delta_1}{2}\right) V_{\mathrm{ds1}}^2}{\left\{1 + \theta_1 \left(V_{\mathrm{gs}} - V_{\mathrm{T1}}\right) + \theta_2 \left(\sqrt{V_{\mathrm{sb}} + \phi_{\mathrm{B}}} - \sqrt{\phi_{\mathrm{B}}}\right)\right\} \left(1 + \theta_3 V_{\mathrm{ds1}}\right)}$$

- drain series resistance is included in θ_3

$$\theta_3 \approx \theta_{30} - \beta R_{\mathrm{d}} - \tfrac{1}{2} \theta_{10}$$

saturation region

- drain series resistance is included in θ_1 and θ_3

$$\theta_1 \; = \; \theta_{10} + \beta\,(R_\mathrm{s} + R_\mathrm{d})$$

$$\theta_3 \; \approx \; \theta_{30} - \beta\,R_\mathrm{d} - \tfrac{1}{2}\,\theta_{10}$$

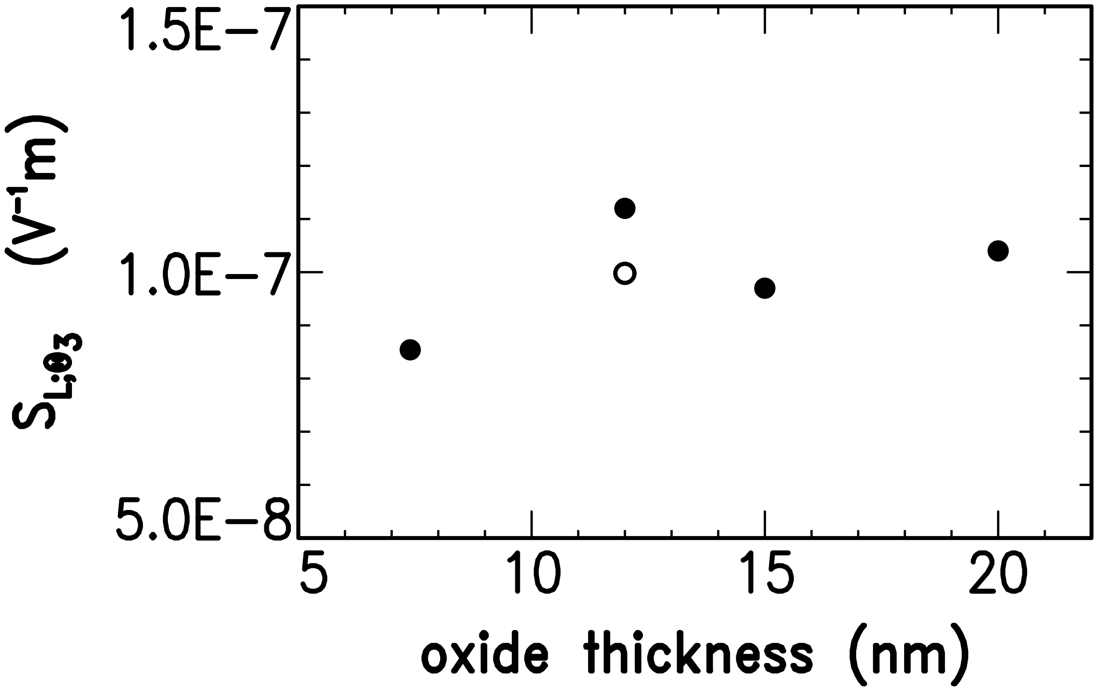
$$\theta_1 + \theta_3 \; \approx \; \tfrac{1}{2}\,\theta_{10} + \theta_{30} + \beta\,R_\mathrm{s}$$

- MM9 expression for I_ds with $V_\mathrm{sb} = 0$

$$I_\mathrm{ds} \; = \; \beta \; \frac{V_\mathrm{gt3}\,V_\mathrm{ds1} - \left(\frac{1+\delta_1}{2}\right)\,V_\mathrm{ds1}^{2}}{\{1 + \theta_1\,(V_\mathrm{gs} - V_\mathrm{T1})\}\,(1 + \theta_3\,V_\mathrm{ds1})}$$

- with $V_\mathrm{ds1} = V_\mathrm{dss1} \approx V_\mathrm{gt3} \approx V_\mathrm{gs} - V_\mathrm{T1} \quad \Rightarrow \quad$ MM9 expression for $I_\mathrm{ds,\,sat}$

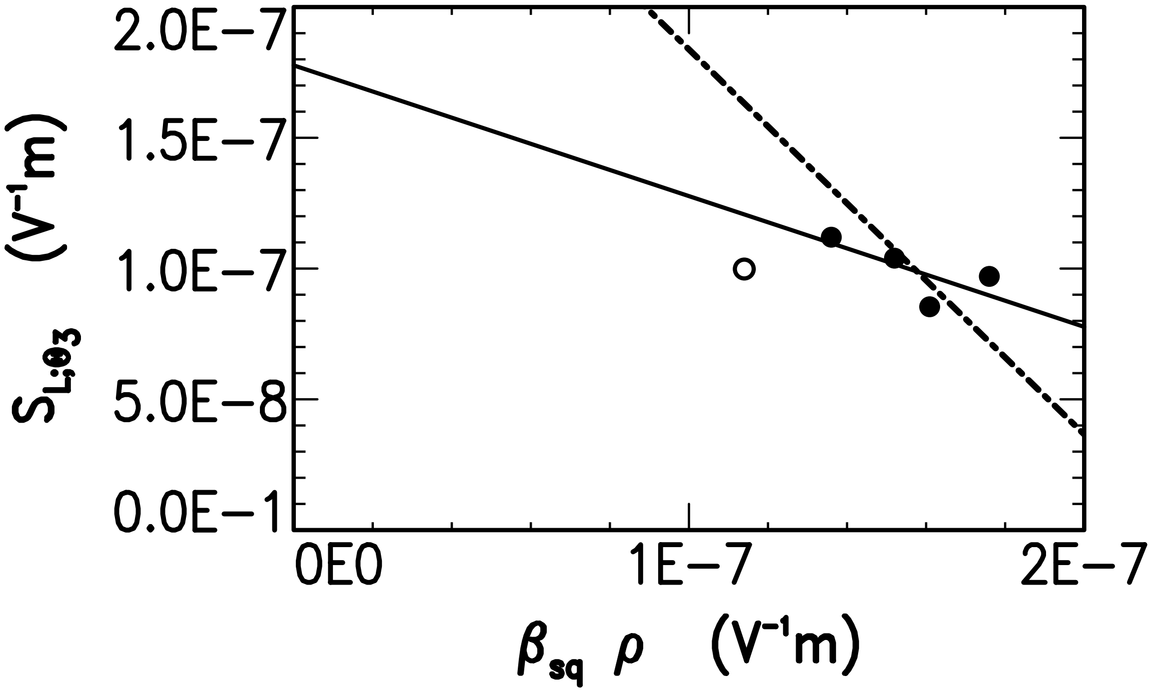
$$I_\mathrm{ds,\,sat} \; \approx \; \beta \; \frac{\left(\frac{1-\delta_1}{2}\right)(V_\mathrm{gs} - V_\mathrm{T1})^{\,2}}{1 + (\theta_1 + \theta_3)\,(V_\mathrm{gs} - V_\mathrm{T1})}$$



- process dependence of the length dependence of θ_3 for N-channels

$$\theta_3 \approx \theta_{30} - \beta R_{\mathrm{d}} - \tfrac{1}{2}\theta_{10} \quad \Rightarrow \quad S_{\mathrm{L};\theta_3,\mathrm{R}} \approx \frac{1}{E_{\mathrm{sat}}} - \frac{\beta_{\square}\,\rho}{2}$$

saturation region



- process dependence of the length dependence of θ_3 for N-channels

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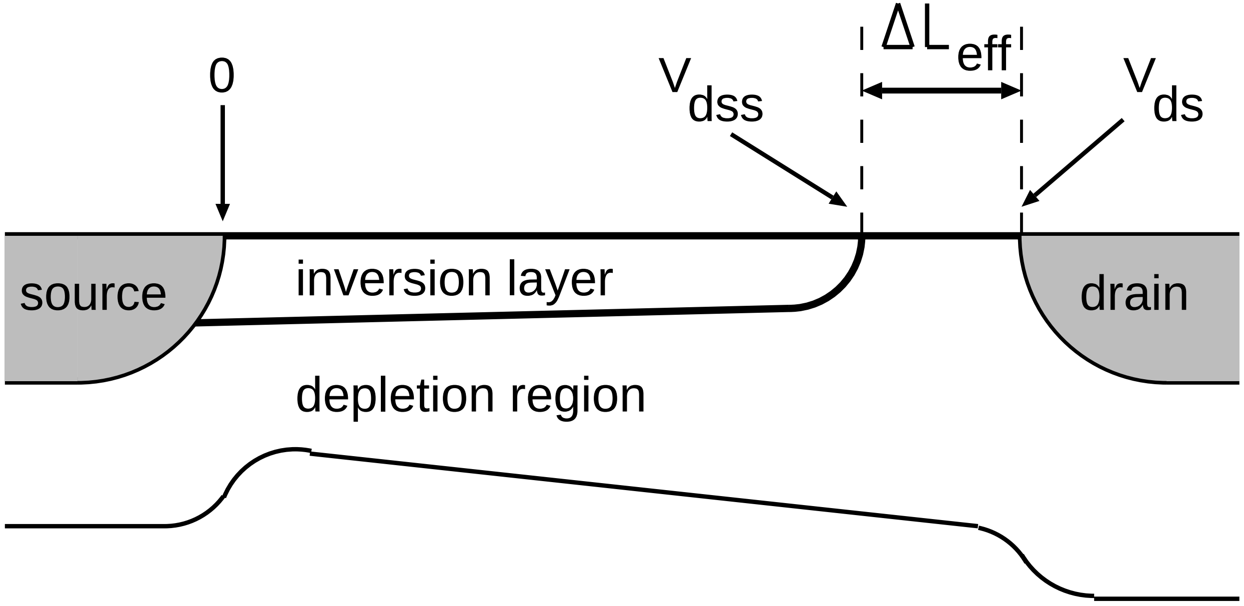
output conductance



- at high drain-source voltages a further increase in drain-source voltage
  - reduces the channel length ⇒ channel-length modulation
  - increases the mobile charge in the channel ⇒ static feedback

output conductance

- channel-length modulation ΔL_{eff}

$$\frac{\Delta L_{\text{eff}}}{L_{\text{eff}}} \approx \alpha \ln\left(1 + \frac{V_{\text{ds}} - V_{\text{ds1}}}{V_{\text{p}}}\right)$$

- channel-length modulation is modelled by multiplying drain current by

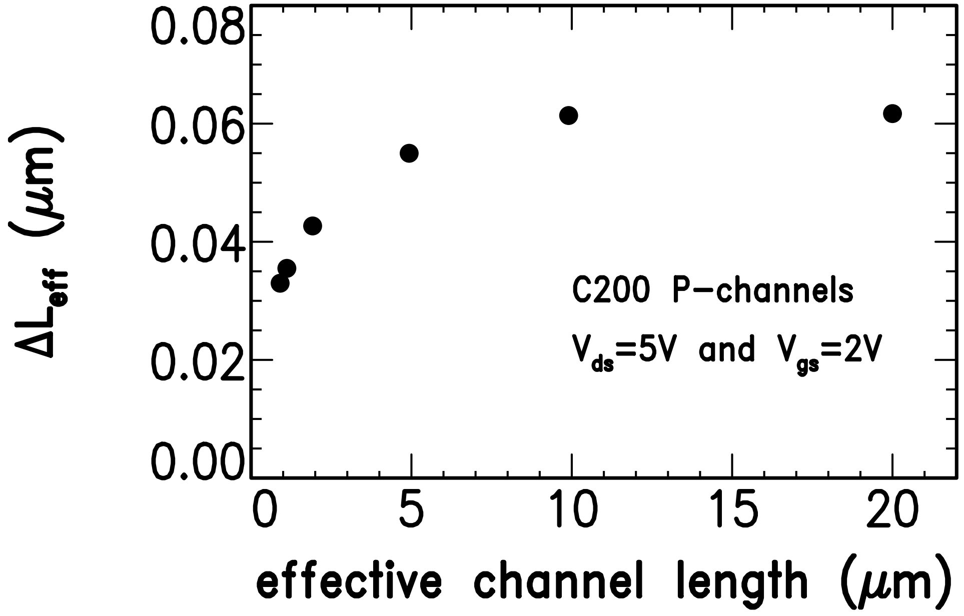
$$G_2 = 1 + \alpha \ln\left(1 + \frac{V_{\text{ds}} - V_{\text{ds1}}}{V_{\text{p}}}\right)$$

- but only in strong inversion by incorporation in G_3

$$G_3 = \frac{\zeta_1 \left\{1 - \exp\left(-\frac{V_{\text{ds}}}{\phi_{\text{T}}}\right)\right\} + G_1 G_2}{\frac{1}{\zeta_1} + G_1}$$

- α and V_{p} are the MM9 parameters for channel-length modulation

output conductance



- for $L_{\text{eff}} > 5\,\mu$m ΔL_{eff} is almost independent of channel length
- for $L_{\text{eff}} < 5\,\mu$m static feedback is dominating!
- multiplication factor G_2 is **not** independent of channel length!

output conductance

- static feedback is modelled by a shift in threshold voltage

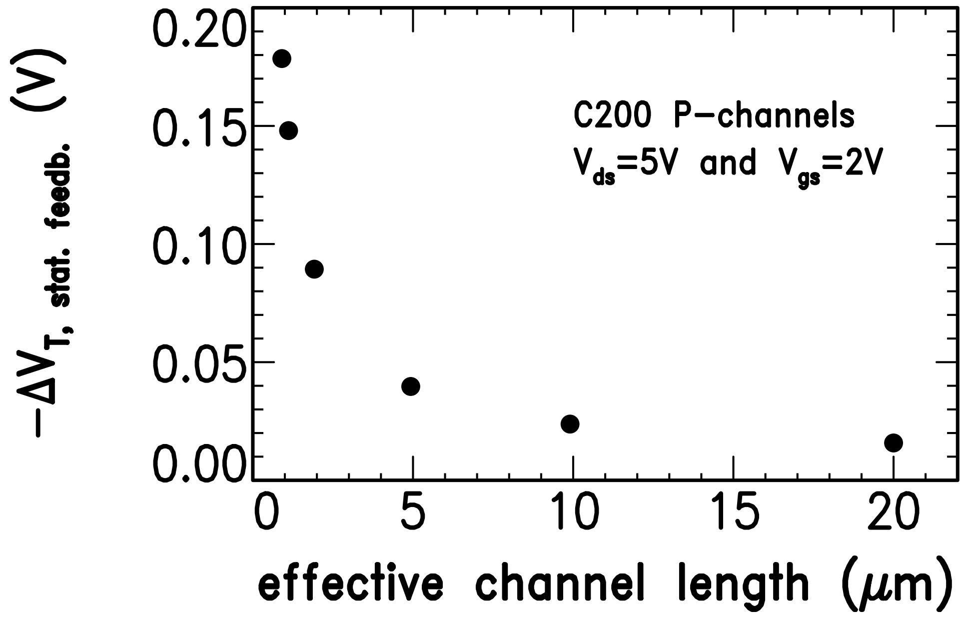
$$\Delta V_{\mathsf{T,\,stat.\,feedb.}} = -\gamma_1 V_{\mathsf{ds}}^{0.6}$$

- static feedback is only effective in strong inversion

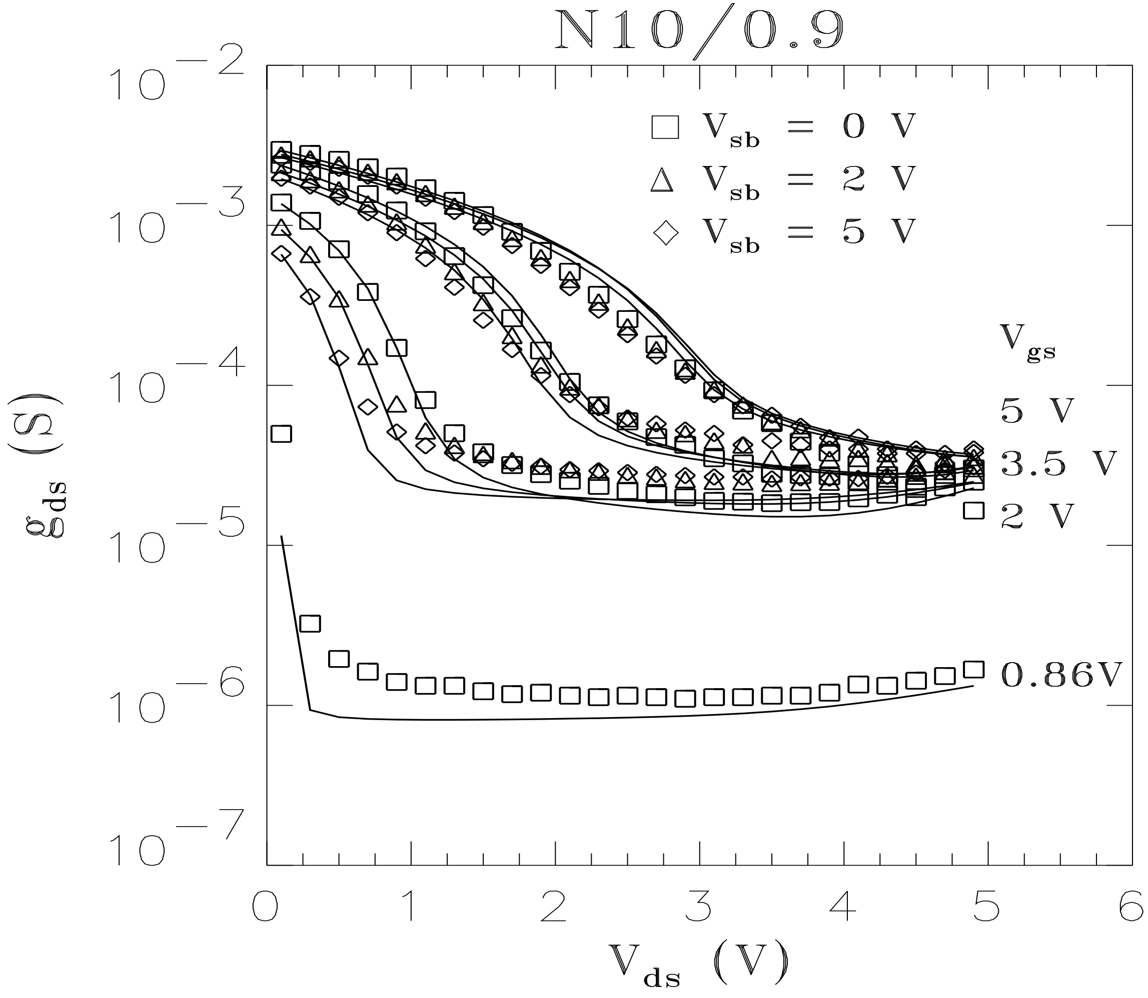
$$\Delta V_{\mathsf{T1}} = \frac{V_{\mathsf{gtx}}^2}{V_{\mathsf{gtx}}^2 + V_{\mathsf{gt1}}^2} \Delta V_{\mathsf{T,\,dibl}} + \frac{V_{\mathsf{gt1}}^2}{V_{\mathsf{gtx}}^2 + V_{\mathsf{gt1}}^2} \Delta V_{\mathsf{T,\,stat.\,feedb.}}$$

- herein $V_{\mathsf{gt1}} = \max\{0, (V_{\mathsf{gs}} - V_{\mathsf{T1}})\}$ and $V_{\mathsf{gtx}} = 0.707V$
- note: $V_{\mathsf{T2}} = V_{\mathsf{T1}} + \Delta V_{\mathsf{T1}}$
- γ_1 is the MM9 parameter for static feedback

output conductance



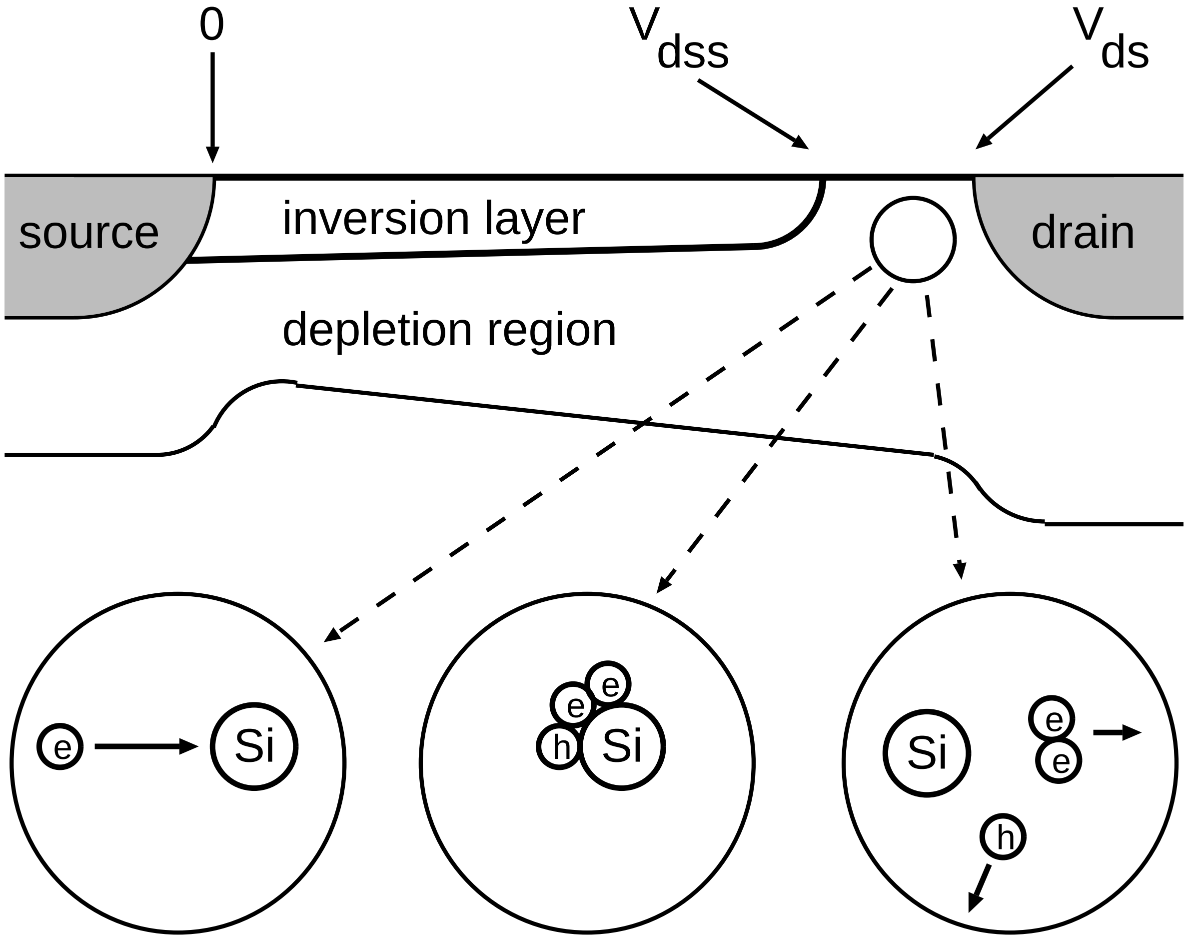
- $\Delta V_{\text{stat. feedb.}}$ depends strongly on channel length
- for $L_{\text{eff}} < 5\,\mu$m static feedback is dominating over channel-length modulation!



MOS MODEL 9

output conductance

substrate current

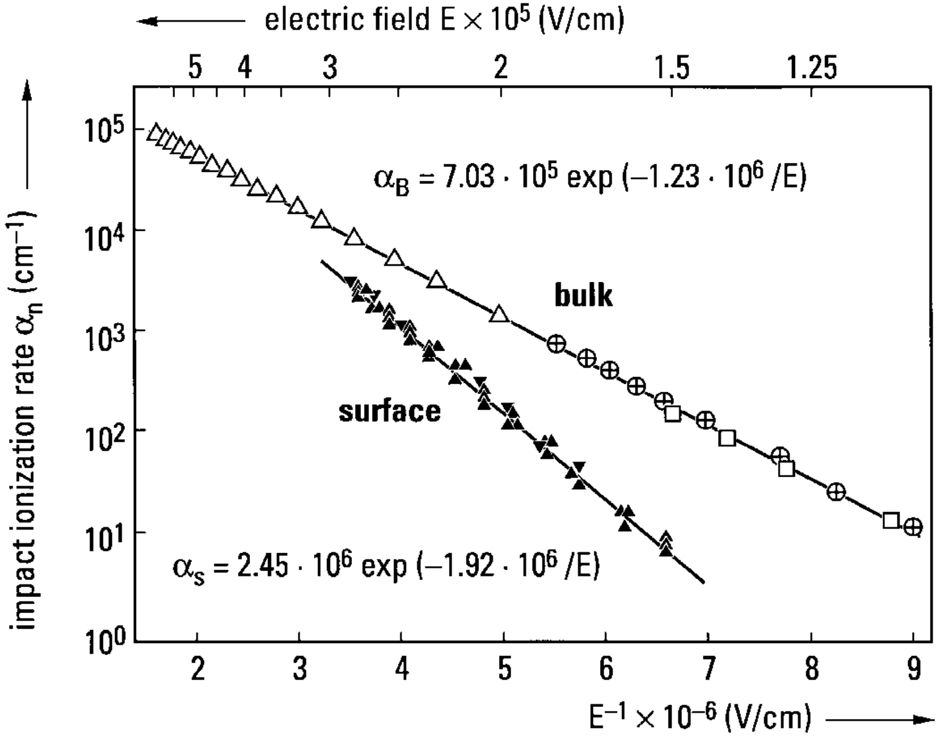


- substrate current is generated by impact-ionization

substrate current

- generation by impact-ionization is given by

$$R_{\mathrm{ii}} = \frac{-1}{q} \left(\alpha_{\mathrm{n}} \mid J_{\mathrm{n}} \mid + \alpha_{\mathrm{p}} \mid J_{\mathrm{p}} \mid \right)$$



impact-ionization rates for electrons vs. $E_{\|}$

substrate current

- impact-ionization varies strongly with electrical field

$$\alpha(E) = A \exp\left(-\frac{B}{E_{\parallel}}\right)$$

- weak avalanche:
  impact-ionization can be modelled as a multiplication factor

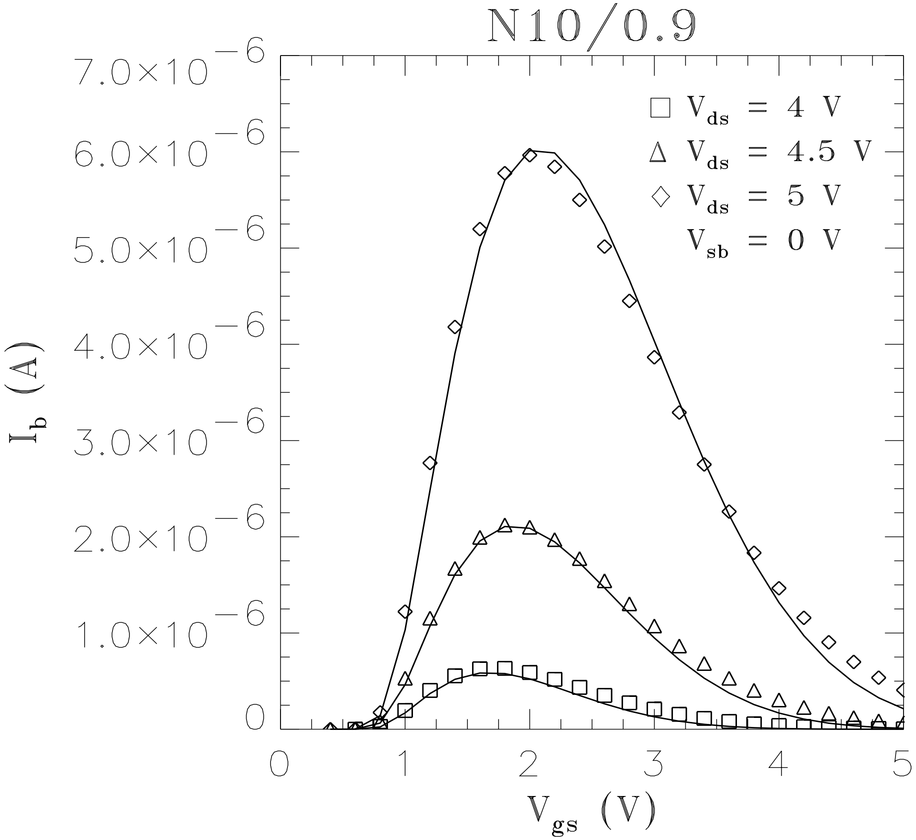
$$I_{\mathrm{avl}} = a_1 I_{\mathrm{ds}} \exp\left(\frac{-a_2}{V_{\mathrm{ds}} - V_{\mathrm{dsa}}}\right)$$

- sufficient high fields only in pinch-off region

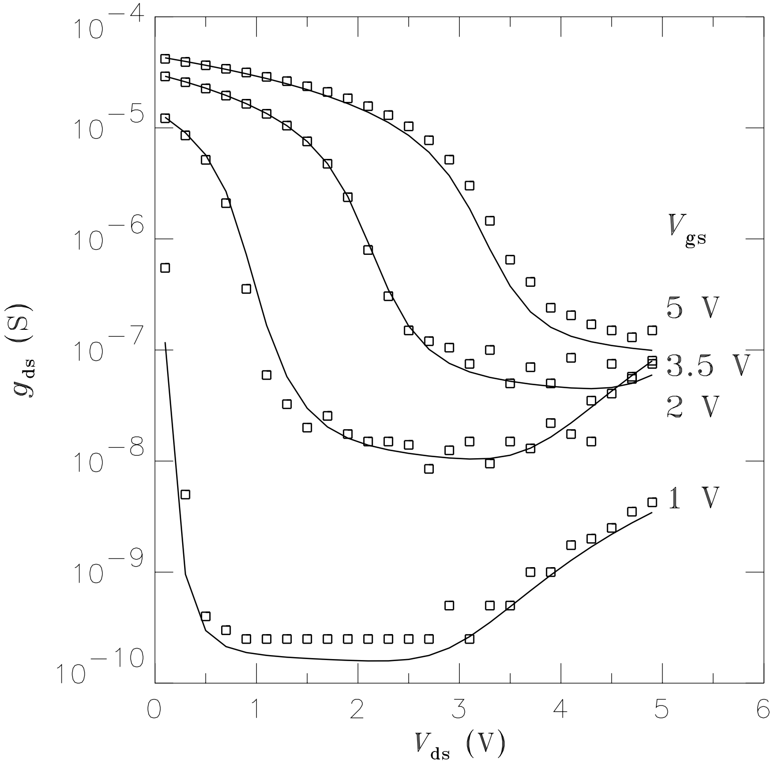
$$V_{\mathrm{dsa}} = a_3 V_{\mathrm{dss1}} \quad \text{and} \quad I_{\mathrm{avl}} = 0 \;\; \text{if} \;\; V_{\mathrm{ds}} < V_{\mathrm{dsa}}$$

- I_{ds} and I_{avl} have to be added for total drain current
- a_1, a_2 and a_3 are the MM9 parameters for avalanche current

substrate current



MOS MODEL 9 substrate current



N1.2/10: effect of substrate current on output conductance

MOS MODEL 9

- relation between physics and equations
- no exact, detailed derivations
- trends in specific parameters
- description and no prediction of measurements

literature:
“Compact Transistor Modelling for Circuit Design”
by H.C. de Graaff and F.M. Klaassen,
Springer-Verlag, Wien-New York, 1990