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Chapter 1
Library

1.1 Introduction

1.1.1 Overview

This user's guide describes the Advanced Motor Control Library (AMCLIB) for the family of DSP56800E core-based digital signal controllers. This library contains optimized functions.

1.1.2 Data types

AMCLIB supports several data types: (un)signed integer, fractional, and accumulator. The integer data types are useful for general-purpose computation; they are familiar to the MPU and MCU programmers. The fractional data types enable powerful numeric and digital-signal-processing algorithms to be implemented. The accumulator data type is a combination of both; that means it has the integer and fractional portions.

The following list shows the integer types defined in the libraries:

- **Unsigned 16-bit integer** — \(0 ; 65535\) with the minimum resolution of 1
- **Signed 16-bit integer** — \(-32768 ; 32767\) with the minimum resolution of 1
- **Unsigned 32-bit integer** — \(0 ; 4294967295\) with the minimum resolution of 1
- **Signed 32-bit integer** — \(-2147483648 ; 2147483647\) with the minimum resolution of 1

The following list shows the fractional types defined in the libraries:

- **Fixed-point 16-bit fractional** — \(-1 ; 1 - 2^{-15}\) with the minimum resolution of \(2^{-15}\)
- **Fixed-point 32-bit fractional** — \(-1 ; 1 - 2^{-31}\) with the minimum resolution of \(2^{-31}\)
The following list shows the accumulator types defined in the libraries:

- **Fixed-point 16-bit accumulator** — \((-256.0 ; 256.0 - 2^{-7}\) with the minimum resolution of \(2^{-7}\)
- **Fixed-point 32-bit accumulator** — \((-65536.0 ; 65536.0 - 2^{-15}\) with the minimum resolution of \(2^{-15}\)

### 1.1.3 API definition

AMCLIB uses the types mentioned in the previous section. To enable simple usage of the algorithms, their names use set prefixes and postfixes to distinguish the functions' versions. See the following example:

```c
f32Result = MLIB_Mac_F32lss(f32Accum, f16Mult1, f16Mult2);
```

where the function is compiled from four parts:

- **MLIB**—this is the library prefix
- **Mac**—the function name—Multiply-Accumulate
- **F32**—the function output type
- **lss**—the types of the function inputs; if all the inputs have the same type as the output, the inputs are not marked

The input and output types are described in the following table:

<table>
<thead>
<tr>
<th>Type</th>
<th>Output</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>frac16_t</td>
<td>F16</td>
<td>s</td>
</tr>
<tr>
<td>frac32_t</td>
<td>F32</td>
<td>l</td>
</tr>
<tr>
<td>acc32_t</td>
<td>A32</td>
<td>a</td>
</tr>
</tbody>
</table>

### 1.1.4 Supported compilers

AMCLIB for the DSP56800E core is written in assembly language with C-callable interface. The library is built and tested using the following compilers:

- **CodeWarrior™ Development Studio**

For the CodeWarrior™ Development Studio, the library is delivered in the `amlib.lib` file.
The interfaces to the algorithms included in this library are combined into a single public interface include file, `amclib.h`. This is done to lower the number of files required to be included in your application.

### 1.1.5 Special issues

1. The equations describing the algorithms are symbolic. If there is positive 1, the number is the closest number to 1 that the resolution of the used fractional type allows. If there are maximum or minimum values mentioned, check the range allowed by the type of the particular function version.

2. The library functions require the core saturation mode to be turned off, otherwise the results can be incorrect. Several specific library functions are immune to the setting of the saturation mode.

3. The library functions round the result (the API contains Rnd) to the nearest (two's complement rounding) or to the nearest even number (convergent round). The mode used depends on the core option mode register (OMR) setting. See the core manual for details.

4. All non-inline functions are implemented without storing any of the volatile registers (refer to the compiler manual) used by the respective routine. Only the non-volatile registers (C10, D10, R5) are saved by pushing the registers on the stack. Therefore, if the particular registers initialized before the library function call are to be used after the function call, it is necessary to save them manually.

### 1.2 Library integration into project (CodeWarrior™ Development Studio)

This section provides a step-by-step guide to quickly and easily integrate the AMCLIB into an empty project using CodeWarrior™ Development Studio. This example uses the MC56F8257 part, and the default installation path (C:\Freescale\FSLESL\DSP56800E_FSLESL_4.2) is supposed. If you have a different installation path, you must use that path instead.

#### 1.2.1 New project

To start working on an application, create a new project. If the project already exists and is open, skip to the next section. Follow the steps given below to create a new project.

2. Choose File > New > Bareboard Project, so that the "New Bareboard Project" dialog appears.
3. Type a name of the project, for example, MyProject01.
4. If you don't use the default location, untick the “Use default location” checkbox, and type the path where you want to create the project folder; for example, C:\CWProjects\MyProject01, and click Next. See Figure 1-1.

![Figure 1-1. Project name and location](image1)

5. Expand the tree by clicking the 56800/E (DSC) and MC56F8257. Select the Application option and click Next. See Figure 1-2.

![Figure 1-2. Processor selection](image2)

6. Now select the connection that will be used to download and debug the application. In this case, select the option P&E USB MultiLink Universal[FX] / USB MultiLink and Freescale USB TAP, and click Next. See Figure 1-3.
Figure 1-3. Connection selection
7. From the options given, select the Simple Mixed Assembly and C language, and click Finish. See Figure 1-4.

Figure 1-4. Language choice

The new project is now visible in the left-hand part of CodeWarrior™ Development Studio. See Figure 1-5.

1.2.2 Library path variable

To make the library integration easier, create a variable that will hold the information about the library path.

1. Right-click the MyProject01 node in the left-hand part and click Properties, or select Project > Properties from the menu. The project properties dialog appears.
2. Expand the Resource node and click Linked Resources. See Figure 1-6.

![Figure 1-6. Project properties](image)

3. Click the 'New...' button on the right-hand side.
4. In the dialog that appears (see Figure 1-7), type this variable name into the Name box: FSLESL_LOC
5. Select the library parent folder by clicking 'Folder...' or just typing the following path into the Location box: C:\Freescale\FSLESL\DSP56800E_FSLESL_4.2_CW and click OK.
6. Click OK in the previous dialog.
1.2.3 Library folder addition

To use the library, add it into the CodeWarrior Project tree dialog.

1. Right-click the MyProject01 node in the left-hand part and click New > Folder, or select File > New > Folder from the menu. A dialog appears.
2. Click Advanced to show the advanced options.
3. To link the library source, select the third option—Link to alternate location (Linked Folder).
4. Click Variables…, and select the FSLESL_LOC variable in the dialog that appears, click OK, and/or type the variable name into the box. See Figure 1-8.
5. Click Finish, and you will see the library folder linked in the project. See Figure 1-9.
1.2.4 Library path setup

AMCLIB requires MLIB and GFLIB and GMCLIB to be included too. Therefore, the following steps show the inclusion of all dependent modules.

1. Right-click the MyProject01 node in the left-hand part and click Properties, or select Project > Properties from the menu. A dialog with the project properties appears.
2. Expand the C/C++ Build node, and click Settings.
3. In the right-hand tree, expand the DSC Linker node, and click Input. See Figure 1-11.
4. In the third dialog Additional Libraries, click the 'Add…' icon, and a dialog appears.
5. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box by adding one of the following:
   - `${FSLESL_LOC}\MLIB\mlib_SDM.lib`—for small data model projects
   - `${FSLESL_LOC}\MLIB\mlib_LDM.lib`—for large data model projects
6. Tick the box Relative To, and select FSLESL_LOC next to the box. See Figure 1-9. Click OK.
7. Click the 'Add…' icon in the third dialog Additional Libraries.
8. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box by adding one of the following:
   - `${FSLESL_LOC}\GFLIB\gflib_SDM.lib`—for small data model projects
   - `${FSLESL_LOC}\GFLIB\gflib_LDM.lib`—for large data model projects
9. Tick the box Relative To, and select FSLESL_LOC next to the box. Click OK.
10. Click the 'Add…' icon in the Additional Libraries dialog.
11. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box by adding one of the following:
    - `${FSLESL_LOC}\GMCLIB\gmclib_SDM.lib`—for small data model projects
    - `${FSLESL_LOC}\GMCLIB\gmclib_LDM.lib`—for large data model projects
12. Tick the box Relative To, and select FSLESL_LOC next to the box. Click OK.
13. Click the 'Add…' icon in the Additional Libraries dialog.
14. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box by adding one of the following:
    - `${FSLESL_LOC}\AMCLIB\amclib_SDM.lib`—for small data model projects
    - `${FSLESL_LOC}\AMCLIB\amclib_LDM.lib`—for large data model projects
15. Now, you will see the libraries added in the box. See Figure 1-11.
16. In the tree under the DSC Compiler node, click Access Paths.

17. In the Search User Paths dialog (#include “…”), click the 'Add...' icon, and a dialog will appear.

18. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box to be: ${FSLESL_LOC}\MLIB\include.

19. Tick the box Relative To, and select FSLESL_LOC next to the box. See Figure 1-12. Click OK.

20. Click the 'Add...' icon in the Search User Paths dialog (#include “…”).

21. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box to be: ${FSLESL_LOC}\GFLIB\include.

22. Tick the box Relative To, and select FSLESL_LOC next to the box. Click OK.

23. Click the 'Add...' icon in the Search User Paths dialog (#include “…”).

24. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box to be: ${FSLESL_LOC}\GMCLIB\include.

25. Tick the box Relative To, and select FSLESL_LOC next to the box. Click OK.

26. Click the 'Add...' icon in the Search User Paths dialog (#include “…”).
27. Look for the FSLESL_LOC variable by clicking Variables…, and then finish the path in the box to be: ${FSLESL_LOC}\AMCLIB\include.

28. Tick the box Relative To, and select FSLESL_LOC next to the box. Click OK.

29. Now you will see the paths added in the box. See Figure 1-13. Click OK.

![Figure 1-12. Library include path addition](image)

The final step is typing the #include syntax into the code. Include the library into the main.c file. In the left-hand dialog, open the Sources folder of the project, and double-click the main.c file. After the main.c file opens up, include the following lines into the #include section:

```c
#include "mlib.h"
#include "gflib.h"
#include "gmclib.h"
#include "amclib.h"
```

When you click the Build icon (hammer), the project will be compiled without errors.

![Figure 1-13. Compiler setting](image)
Chapter 2
Algorithms in detail

2.1 AMCLIB_TrackObsrv

The AMCLIB_TrackObsrv function calculates a tracking observer for the determination of angular speed and position of the input error functional signal. The tracking-observer algorithm uses the phase-locked-loop mechanism. It is recommended to call this function at every sampling period. It requires a single input argument as a phase error. A phase-tracking observer with a standard PI controller used as the loop compensator is shown in Figure 2-1.

The depicted tracking observer structure has the following transfer function:

\[
\frac{\hat{\theta}(s)}{\theta(s)} = \frac{sK_p + K_i}{s^2 + sK_p + K_f}
\]

Equation 1

The controller gains \(K_p\) and \(K_i\) are calculated by comparing the characteristic polynomial of the resulting transfer function to a standard second-order system polynomial.

The essential equations for implementation of the tracking observer according to the block scheme in Figure 2-1 are as follows:
where:

- $K_P$ is the proportional gain
- $K_I$ is the integral gain
- $T_s$ is the sampling period [s]
- $e(k)$ is the position error in step $k$
- $\omega(k)$ is the rotor speed [rad / s] in step $k$
- $\omega(k-1)$ is the rotor speed [rad / s] in step $k-1$
- $\theta(k)$ is the rotor angle [rad] in step $k$
- $\theta(k-1)$ is the rotor angle [rad] in step $k-1$

In the fractional arithmetic, Equation 1 on page 17 and Equation 2 on page 18 are as follows:

$$\omega_{sc}(k) \cdot \omega_{\text{max}} = K_P \cdot e_{sc}(k) \cdot \omega_{\text{max}} + T_s \cdot K_I \cdot e_{sc}(k) \cdot \theta_{\text{max}} + \omega_{sc}(k-1) \cdot \omega_{\text{max}}$$

Equation 4

$$\theta_{sc}(k) \cdot \theta_{\text{max}} = T_s \cdot \omega_{sc}(k) \cdot \omega_{\text{max}} + \theta_{sc}(k-1) \cdot \theta_{\text{max}}$$

Equation 5

where:

- $e_{sc}(k)$ is the scaled position error in step $k$
- $\omega_{sc}(k)$ is the scaled rotor speed [rad / s] in step $k$
- $\omega_{sc}(k-1)$ is the scaled rotor speed [rad / s] in step $k-1$
- $\theta_{sc}(k)$ is the scaled rotor angle [rad] in step $k$
- $\theta_{sc}(k-1)$ is the scaled rotor angle [rad] in step $k-1$
- $\omega_{\text{max}}$ is the maximum speed
- $\theta_{\text{max}}$ is the maximum rotor angle (typically)

### 2.1.1 Available versions

The function is available in the following versions:

- Fractional output - the output is the fractional portion of the result; the result is within the range $<-1 ; 1>$. 

---

Equation 2

$$\theta(k) = T_s \cdot \omega(k) + \theta(k-1)$$

Equation 3
The available versions of the `AMCLIB_TrackObsrv` function are shown in the following table:

### Table 2-1. Init versions

<table>
<thead>
<tr>
<th>Function name</th>
<th>Init angle</th>
<th>Parameters</th>
<th>Result type</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCLIB_TrackObsrvInit_F16</td>
<td>frac16_t</td>
<td>AMCLIB_TRACK_OBSRV_T_F32 *</td>
<td>void</td>
</tr>
</tbody>
</table>

The input is a 16-bit fractional value of the angle normalized to the range $<-1 ; 1)$ that represents an angle (in radians) within the range $<-\pi ; \pi)$.

### Table 2-2. Function versions

<table>
<thead>
<tr>
<th>Function name</th>
<th>Input type</th>
<th>Parameters</th>
<th>Result type</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCLIB_TrackObsrv_F16</td>
<td>frac16_t</td>
<td>AMCLIB_TRACK_OBSRV_T_F32 *</td>
<td>frac16_t</td>
</tr>
</tbody>
</table>

Tracking observer with a 16-bit fractional position error input divided by $\pi$. The output from the observer is a 16-bit fractional position normalized to the range $<-1 ; 1)$ that represents an angle (in radians) within the range $<-\pi ; \pi)$.

#### 2.1.2 AMCLIB_TRACK_OBSRV_T_F32

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Input type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>f32Theta</td>
<td>frac32_t</td>
<td>Estimated position as the output of the second numerical integrator. The parameter is within the range $&lt;-1 ; 1)$. Controlled by the algorithm.</td>
</tr>
<tr>
<td>f32Speed</td>
<td>frac32_t</td>
<td>Estimated speed as the output of the first numerical integrator. The parameter is within the range $&lt;-1 ; 1)$. Controlled by the algorithm.</td>
</tr>
<tr>
<td>f32I_1</td>
<td>frac32_t</td>
<td>State variable in the controller part of the observer; integral part at step $k - 1$. The parameter is within the range $&lt;-1 ; 1)$. Controlled by the algorithm.</td>
</tr>
<tr>
<td>f16IGain</td>
<td>frac16_t</td>
<td>The observer integral gain is set up according to Equation 4 on page 18 as: $T_s \cdot K_f \cdot g_{\frac{0_{\text{max}}}{0_{\text{max}}} \cdot 2^{-ISH}}$ The parameter is a 16-bit fractional type within the range $&lt;0 ; 1)$. Set by the user.</td>
</tr>
<tr>
<td>i16IGainSh</td>
<td>int16_t</td>
<td>The observer integral gain shift takes care of keeping the f16IGain variable within the fractional range $&lt;-1 ; 1)$. The shift is determined as: $\log_2(T_s \cdot K_f \cdot g_{\frac{0_{\text{max}}}{0_{\text{max}}} \cdot 2^{-ISH}}) \leq \log_2(0.5)$ The parameter is a 16-bit integer type within the range $&lt;-15 ; 15&gt;$. Set by the user.</td>
</tr>
<tr>
<td>f16PGain</td>
<td>frac16_t</td>
<td>The observer proportional gain is set up according to Equation 4 on page 18 as: $K_p \cdot g_{\frac{0_{\text{max}}}{0_{\text{max}}} \cdot 2^{-PSH}}$ The parameter is a 16-bit fractional type within the range $&lt;0 ; 1)$. Set by the user.</td>
</tr>
<tr>
<td>i16PGainSh</td>
<td>int16_t</td>
<td>The observer proportional gain shift takes care of keeping the f16PGain variable within the fractional range $&lt;-1 ; 1)$. The shift is determined as:</td>
</tr>
</tbody>
</table>

*Table continues on the next page...*
<table>
<thead>
<tr>
<th>Variable name</th>
<th>Input type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>f16ThGain</td>
<td>frac16_t</td>
<td>The observer gain for the output position integrator is set up according to Equation 5 on page 18 as: ( T_s \frac{a_{\text{max}}}{a_{\text{sh}}} - \frac{\gamma}{\text{Thsh}} ). The parameter is a 16-bit fractional type within the range &lt;0; 1). Set by the user.</td>
</tr>
<tr>
<td>i16ThGainSh</td>
<td>int16_t</td>
<td>The observer gain shift for the position integrator takes care of keeping the f16ThGain variable within the fractional range &lt;-1; 1). The shift is determined as: ( \log_2(T_s \frac{a_{\text{max}}}{a_{\text{sh}}} - \frac{\gamma}{\text{Thsh}}) \leq \log_2(T_s \frac{a_{\text{max}}}{a_{\text{sh}}} - \gamma_{\text{sh}}) ). The parameter is a 16-bit integer type within the range &lt;-15; 15&gt;. Set by the user.</td>
</tr>
</tbody>
</table>

### 2.1.3 Declaration

The available AMCLIB_TrackObsrvInit functions have the following declarations:

```c
void AMCLIB_TrackObsrvInit_F16(frac16_t f16ThetaInit, AMCLIB_TRACK_OBSRV_T_F32 *psCtrl)
```

The available `AMCLIB_TrackObsrv` functions have the following declarations:

```c
frac16_t AMCLIB_TrackObsrv_F16(frac16_t f16Error, AMCLIB_TRACK_OBSRV_T_F32 *psCtrl)
```

### 2.1.4 Function use

The use of the `AMCLIB_TrackObsrv` function is shown in the following example:

```c
#include "amclib.h"

static AMCLIB_TRACK_OBSRV_T_F32 sTo;
static frac16_t      f16ThetaError;
static frac16_t      f16PositionEstim;

void Isr(void);

void main(void)
{
  sTo.f16IGain     = FRAC16(0.6434);
  sTo.i16IGainSh   = -9;
  sTo.f16PGain     = FRAC16(0.6801);
  sTo.i16PGainSh   = -2;
  sTo.f16ThGain    = FRAC16(0.6400);
  sTo.i16ThGainSh  = -4;

  AMCLIB_TrackObsrvInit_F16(FRAC16(0.0), &sTo);

  f16ThetaError    = FRAC16(0.5);
}
2.2 AMCLIB_AngleTrackObsrv

The AMCLIB_TrackObsrv function calculates an angle-tracking observer for
determination of angular speed and position of the input signal. It requires two input
arguments as sine and cosine samples. The practical implementation of the angle-tracking
observer algorithm is described below.

The angle-tracking observer compares values of the input signals - sin(θ), cos(θ) with
their corresponding estimations. As in any common closed-loop systems, the intent is to
minimize the observer error towards zero value. The observer error is given here by
subtracting the estimated resolver rotor angle from the actual rotor angle.

The tracking-observer algorithm uses the phase-locked loop mechanism. It is
recommended to call this function at every sampling period. It requires a single input
argument as phase error. A phase-tracking observer with standard PI controller used as
the loop compensator is shown in Figure 2-2.

![Figure 2-2. Block diagram of proposed PLL scheme for position estimation](image-url)

Note that the mathematical expression of the observer error is known as the formula of
the difference between two angles:
If the deviation between the estimated and the actual angle is very small, then the observer error may be expressed using the following equation:

$$\sin(\theta - \hat{\theta}) = \sin(\theta) \cdot \cos(\hat{\theta}) - \cos(\theta) \cdot \sin(\hat{\theta})$$

**Equation 6**

The primary benefit of the angle-tracking observer utilization, in comparison with the trigonometric method, is its smoothing capability. This filtering is achieved by the integrator and the proportional and integral controllers, which are connected in series and closed by a unit feedback loop. This block diagram tracks the actual rotor angle and speed, and continuously updates their estimations. The angle-tracking observer transfer function is expressed as follows:

$$\frac{\hat{\theta}(s)}{\theta(s)} = \frac{K_p(1+K_i)}{s^2 + sK_pK_i + K_i}$$

**Equation 8**

The characteristic polynomial of the angle-tracking observer corresponds to the denominator of the following transfer function:

$$s^2 + sK_pK_i + K_i$$

Appropriate dynamic behavior of the angle-tracking observer is achieved by the placement of the poles of characteristic polynomial. This general method is based on matching the coefficients of characteristic polynomial with the coefficients of a general second-order system.

The analog integrators in the previous figure (marked as $1/s$) are replaced by an equivalent of the discrete-time integrator using the backward Euler integration method. The discrete-time block diagram of the angle-tracking observer is shown in the following figure:
The essential equations for implementing the angle-tracking observer (according to this block scheme) are as follows:

\[ e(k) = \sin(\theta(k)) \cdot \cos(\dot{\theta}(k - 1)) - \cos(\theta(k)) \cdot \sin(\dot{\theta}(k - 1)) \]

**Equation 9**

\[ \omega(k) = T_s \cdot K_1 \cdot e(k) + \omega(k - 1) \]

**Equation 10**

\[ a_2(k) = T_s \cdot \omega(k) + a_2(k - 1) \]

**Equation 11**

\[ \hat{\theta}(k) = K_2 \cdot \omega(k) + a_2(k) \]

**Equation 12**

where:

- \( K_1 \) is the integral gain of the I controller
- \( K_2 \) is the proportional gain of the PI controller
- \( T_s \) is the sampling period [s]
- \( e(k) \) is the position error in step k
- \( \omega(k) \) is the rotor speed [rad / s] in step k
- \( \omega(k - 1) \) is the rotor speed [rad / s] in step k - 1
- \( a(k) \) is the integral output of the PI controller [rad / s] in step k
- \( a(k - 1) \) is the integral output of the PI controller [rad / s] in step k - 1
- \( \theta(k) \) is the rotor angle [rad] in step k
- \( \theta(k - 1) \) is the rotor angle [rad] in step k - 1
• \( \hat{\theta}(k) \) is the estimated rotor angle [rad] in step \( k \)
• \( \hat{\theta}(k - 1) \) is the estimated rotor angle [rad] in step \( k - 1 \)

In the fractional arithmetic, Equation 9 on page 23 to Equation 12 on page 23 are as follows:

\[
\omega_{sc}(k) \cdot \omega_{max} = T_s \cdot K_1 \cdot e(k) + \omega_{sc}(k - 1) \cdot \omega_{max}
\]

**Equation 13**

\[
a_{2sc}(k) \cdot \theta_{max} = T_s \cdot \omega_{sc}(k) \cdot \omega_{max} + a_{2sc}(k - 1) \cdot \theta_{max}
\]

**Equation 14**

\[
\dot{\theta}_{sc}(k) \cdot \theta_{max} = K_2 \cdot \omega_{sc}(k) \cdot \omega_{max} + a_{2sc}(k) \cdot \theta_{max}
\]

**Equation 15**

where:

• \( e_{sc}(k) \) is the scaled position error in step \( k \)
• \( \omega_{sc}(k) \) is the scaled rotor speed [rad / s] in step \( k \)
• \( \omega_{sc}(k - 1) \) is the scaled rotor speed [rad / s] in step \( k - 1 \)
• \( a_{sc}(k) \) is the integral output of the PI controller [rad / s] in step \( k \)
• \( a_{sc}(k - 1) \) is the integral output of the PI controller [rad / s] in step \( k - 1 \)
• \( \theta_{sc}(k) \) is the scaled rotor angle [rad] in step \( k \)
• \( \theta_{sc}(k - 1) \) is the scaled rotor angle [rad] in step \( k - 1 \)
• \( \omega_{max} \) is the maximum speed
• \( \theta_{max} \) is the maximum rotor angle (typically \( \pi \))

### 2.2.1 Available versions

The function is available in the following versions:

• Fractional output - the output is the fractional portion of the result; the result is within the range \(-1 \); 1\).
The available versions of the `AMCLIB_AngleTrackObsrv` function are shown in the following table:

### Table 2-3. Init versions

<table>
<thead>
<tr>
<th>Function name</th>
<th>Init angle</th>
<th>Parameters</th>
<th>Result type</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCLIB_AngleTrackObsrvInit_F16</td>
<td>frac16_t</td>
<td>AMCLIB_ANGLE_TRACK_OBSRV_T_F32 *</td>
<td>void</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The input is a 16-bit fractional value of the angle normalized to the range &lt;-1 ; 1) that represents an angle in (radians) within the range &lt;-π ; π).</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2-4. Function versions

<table>
<thead>
<tr>
<th>Function name</th>
<th>Input type</th>
<th>Parameters</th>
<th>Result type</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCLIB_AngleTrackObsrv_F16</td>
<td>GMCLIB_2COOR_SINCOS_T_F16</td>
<td>AMCLIB_ANGLE_TRACK_OBSRV_T_F32 *</td>
<td>frac16_t</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Angle-tracking observer with a two-component (sin/cos) 16-bit fractional position input within the range &lt;-1 ; 1). The output from the observer is a 16-bit fractional position normalized to the range &lt;-1 ; 1) that represents an angle (in radians) within the range &lt;-π ; π).</td>
<td></td>
</tr>
</tbody>
</table>

### 2.2.2 AMCLIB_ANGLE_TRACK_OBSRV_T_F32

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Input type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>f32Speed</td>
<td>frac32_t</td>
<td>Estimated speed as the output of the first numerical integrator. The parameter is within the range &lt;-1 ; 1). Controlled by the AMCLIB_AngleTrackObsrv_F16 algorithm; cleared by the AMCLIB_AngleTrackObsrvInit_F16 function.</td>
</tr>
<tr>
<td>f32A2</td>
<td>frac32_t</td>
<td>Output of the second numerical integrator. The parameter is within the range &lt;-1 ; 1). Controlled by the AMCLIB_AngleTrackObsrv_F16 and AMCLIB_AngleTrackObsrvInit_F16 algorithms.</td>
</tr>
<tr>
<td>f16Theta</td>
<td>frac16_t</td>
<td>Estimated position as the output of the observer. The parameter is normalized to the range &lt;-1 ; 1) that represents an angle (in radians) within the range &lt;-π ; π). Controlled by the AMCLIB_AngleTrackObsrv_F16 and AMCLIB_AngleTrackObsrvInit_F16 algorithms.</td>
</tr>
<tr>
<td>f16SinEstim</td>
<td>frac16_t</td>
<td>Sine of the estimated position as the output of the actual step. Keeps the sine of the position for the next step. The parameter is within the range &lt;-1 ; 1). Controlled by the AMCLIB_AngleTrackObsrv_F16 and AMCLIB_AngleTrackObsrvInit_F16 algorithms.</td>
</tr>
<tr>
<td>f16CosEstim</td>
<td>frac16_t</td>
<td>Cosine of the estimated position as the output of the actual step. Keeps the cosine of the position for the next step. The parameter is within the range &lt;-1 ; 1). Controlled by the AMCLIB_AngleTrackObsrv_F16 and AMCLIB_AngleTrackObsrvInit_F16 algorithms.</td>
</tr>
<tr>
<td>f16K1Gain</td>
<td>frac16_t</td>
<td>Observer K1 gain is set up according to Equation 13 on page 24 as: $T_s \cdot K_1 \cdot \frac{1}{\omega_{max}} \cdot 2^{-K_{1sh}}$. The parameter is a 16-bit fractional type within the range &lt;0 ; 1). Set by the user.</td>
</tr>
</tbody>
</table>
### 2.2.3 Declaration

The available AMCLIB_AngleTrackObsrvInit functions have the following declarations:

```c
void AMCLIB_AngleTrackObsrvInit_F16(frac16_t f16ThetaInit, AMCLIB_ANGLE_TRACK_OBSRV_T_F32 *psCtrl)
```

The available AMCLIB_AngleTrackObsrv functions have the following declarations:

```c
frac16_t AMCLIB_AngleTrackObsrv_F16(const GMCLIB_2COOR_SINCOS_T_F16 *psAnglePos, AMCLIB_ANGLE_TRACK_OBSRV_T_F32 *psCtrl)
```

### 2.2.4 Function use

The use of the AMCLIB_AngleTrackObsrvInit and AMCLIB_AngleTrackObsrv functions is shown in the following example:
```c
#include "amclib.h"

static AMCLIB_ANGLE_TRACK_OBSRV_T_F32  sAto;
static GMCLIB_2COOR_SINCOS_T_F16 sAnglePos;
static frac16_t      f16PositionEstim, f16PositionInit;

void Isr(void);
void main(void)
{
    void main(void)
    {
    
sAto.f16K1Gain    = FRAC16(0.6434);
sAto.i16K1GainSh  = -9;
sAto.f16K2Gain    = FRAC16(0.6801);
sAto.i16K2GainSh  = -2;
sAto.f16A2Gain    = FRAC16(0.6400);
sAto.i16A2GainSh  = -4;

    f16PositionInit = FRAC16(0.0);

    AMCLIB_AngleTrackObsrvInit_F16(f16PositionInit, &sAto);

    sAnglePos.f16Sin  = FRAC16(0.0);
sAnglePos.f16Cos  = FRAC16(1.0);
    }

    /* Periodical function or interrupt */
    void Isr(void)
    {
    
    /* Angle tracking observer calculation */
    
f16PositionEstim = AMCLIB_AngleTrackObsrv_F16(&sAnglePos, &sAto);
    }
```
The back-EMF observer detects the generated motor voltages induced by the permanent magnets. A tracking observer uses the back-EMF signals to calculate the position and speed of the rotor. The transformed model is then derived as follows:

\[
\begin{bmatrix}
  u_d \\
  u_q \\
\end{bmatrix} = \begin{bmatrix}
  R_s + sL_D & -\omega_rL_Q \\
  \omega_rL_Q & R_s + sL_D \\
\end{bmatrix} \begin{bmatrix}
  i_d \\
  i_q \\
\end{bmatrix} + \begin{bmatrix}
  (\omega_r i_d - s i_q) + \Psi_m \omega_r \\
  \cos(\theta_{\text{error}}) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  u_d \\
  u_q \\
\end{bmatrix} = \begin{bmatrix}
  R_s + sL_D & -\omega_rL_Q \\
  \omega_rL_Q & R_s + sL_D \\
\end{bmatrix} \begin{bmatrix}
  i_d \\
  i_q \\
\end{bmatrix} + \begin{bmatrix}
  (\omega_r i_d - s i_q) + \Psi_m \omega_r \\
  \cos(\theta_{\text{error}}) \\
\end{bmatrix}
\]

\text{Equation 16}

where:

- \( R_s \) is the stator resistance
- \( L_D \) and \( L_Q \) are the D-axis and Q-axis inductances
- \( \Psi_m \) is the back-EMF constant
- \( \omega_r \) is the angular electrical rotor speed
- \( u_Y \) and \( u_S \) are the estimated stator voltages
- \( i_Y \) and \( i_S \) are the estimated stator currents
- \( \theta_{\text{error}} \) is the error between the actual D-Q frame and the estimated frame position
- \( s \) is the operator of the derivative

The block diagram of the observer in the estimated reference frame is shown in Figure 2-5. The observer compensator is substituted by a standard PI controller. As shown in Figure 2-5, the observer model and hence also the PI controller gains in both axes are identical to each other.
Figure 2-5. Block diagram of proposed Luenberger-type stator current observer acting as state filter for back-EMF

The position estimation can now be performed by extracting the $\theta_{\text{error}}$ term from the model, and adjusting the position of the estimated reference frame to achieve $\theta_{\text{error}} = 0$. Because the $\theta_{\text{error}}$ term is only included in the saliency-based EMF component of both $u_\gamma$ and $u_\delta$ axis voltage equations, the Luenberger-based disturbance observer is designed to observe the $u_\gamma$ and $u_\delta$ voltage components. The position displacement information $\theta_{\text{error}}$ is then obtained from the estimated back-EMFs as follows:

$$\theta_{\text{error}} = \text{atan} \left( \frac{-e_\gamma}{e_\delta} \right)$$

Equation 17

The estimated position $\hat{\theta}$ can be obtained by driving the position of the estimated reference mechanism to achieve zero displacement $\theta_{\text{error}} = 0$. The phase-locked-loop mechanism can be adopted, where the loop compensator ensures correct tracking of the actual rotor flux position by keeping the error signal $\theta_{\text{error}}$ zeroed, $\theta_{\text{error}} = 0$.

A perfect match between the actual and estimated motor model parameters is assumed, and then the back-EMF transfer function can be simplified as follows:
The appropriate dynamic behavior of the back-EMF observer is achieved by the placement of the poles of the stator current observer characteristic polynomial. This general method is based on matching the coefficients of the characteristic polynomial with the coefficients of the general second-order system.

The back-EMF observer is a Luenberger-type observer with a motor model, which is implemented using the backward Euler transformation as follows:

$$\dot{E}_{af}(s) = -E_{af}(s) \cdot \frac{F(s)}{sL_D + R_S + F(s)}$$

Equation 18

where:

- $$i(k) = [i_y, i_\delta]$$ is the stator current vector in the actual step
- $$i(k - 1) = [i_y, i_\delta]$$ is the stator current vector in the previous step
- $$u(k) = [u_y, u_\delta]$$ is the stator voltage vector in the actual step
- $$e(k) = [e_y, e_\delta]$$ is the stator back-EMF voltage vector in the actual step
- $$i'(k) = [i_y, -i_\delta]$$ is the complementary stator current vector in the actual step
- $$\omega_e(k)$$ is the electrical angular speed in the actual step
- $$T_S$$ is the sampling time [s]

This equation is transformed into the fractional arithmetic as follows:

$$i_{sc}(k) \cdot i_{max} = \frac{T_s}{L_D + T_s R_S} \cdot u_{sc}(k) \cdot u_{max} + \frac{T_s}{L_D + T_s R_S} \cdot e_{sc}(k) \cdot e_{max} + \frac{L_0 T_s}{L_D + T_s R_S} \cdot \omega_{esc}(k) \cdot \omega_{max} \cdot i'_{sc}(k) \cdot i_{max} + \frac{L_D}{L_D + T_s R_S} \cdot i_{sc}(k - 1) \cdot i_{max}$$

Equation 19

where:

- $$i_{sc}(k) = [i_y, i_\delta]$$ is the scaled stator current vector in the actual step
- $$i_{sc}(k - 1) = [i_y, i_\delta]$$ is the scaled stator current vector in the previous step
- $$u_{sc}(k) = [u_y, u_\delta]$$ is the scaled stator voltage vector in the actual step
- $$e_{sc}(k) = [e_y, e_\delta]$$ is the scaled stator back-EMF voltage vector in the actual step
- $$i'_{sc}(k) = [i_y, -i_\delta]$$ is the scaled complementary stator current vector in the actual step
- $$\omega_{esc}(k)$$ is the scaled electrical angular speed in the actual step
- $$i_{max}$$ is the maximum current [A]
- $$e_{max}$$ is the maximum back-EMF voltage [V]
- $$u_{max}$$ is the maximum stator voltage [V]
- $$\omega_{max}$$ is the maximum electrical angular speed in [rad / s]
If the Luenberger-type stator current observer is properly designed in the stationary reference frame, the back-EMF can be estimated as a disturbance produced by the observer controller. However, this is only valid when the back-EMF term is not included in the observer model. The observer is a closed-loop current observer, therefore it acts as a state filter for the back-EMF term.

The estimate of the extended EMF term can be derived from Equation 18 on page 30 as follows:

\[
-\frac{\hat{E}_d(s)}{E_d(s)} = \frac{sK_p + K_i}{sL_d + sR_s + sK_p + K_i}
\]

**Equation 21**

The observer controller can be designed by comparing the closed-loop characteristic polynomial with that of a standard second-order system as follows:

\[
s^2 + \frac{K_p + R_s}{L_d} \cdot s + \frac{K_i}{L_d} = s^2 + 2\xi\omega_0 s + \omega_0^2
\]

**Equation 22**

where:
- \(\omega_0\) is the natural frequency of the closed-loop system (loop bandwidth)
- \(\xi\) is the loop attenuation
- \(K_p\) is the proportional gain
- \(k_i\) is the integral gain

### 2.3.1 Available versions

This function is available in the following versions:

- Fractional output - the output is the fractional portion of the result; the result is within the range \((-1 ; 1)\). The parameters use the accumulator types.
- Accumulator output with floating-point inputs - the output is the accumulator result; the result is within the range \((-1 ; 1)\). The inputs are 32-bit single precision floating-point values.

The available versions of the AMCLIB_PMSMBemfObsrvDQ function are shown in the following table:

<table>
<thead>
<tr>
<th>Function name</th>
<th>Parameters</th>
<th>Result type</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCLIB_PMSMBemfObsrvDQInit_F16</td>
<td>AMCLIB_BEMF_OBSRV_DQ_T_A32 *</td>
<td>void</td>
</tr>
<tr>
<td>Initialization does not have any input.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 2-6. Function versions

<table>
<thead>
<tr>
<th>Function name</th>
<th>Input/output type</th>
<th>Result type</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCLIB_PMSMBemfObsrvDQ_F16</td>
<td>Input</td>
<td>frac16_t</td>
</tr>
<tr>
<td></td>
<td>GMCLIB_2COOR_DQ_T_F16 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GMCLIB_2COOR_DQ_T_F16 *</td>
<td></td>
</tr>
<tr>
<td></td>
<td>frac16_t</td>
<td></td>
</tr>
<tr>
<td>Parameters</td>
<td>AMCLIB_BEMF_OBSRV_DQ_T_A32 *</td>
<td></td>
</tr>
<tr>
<td>Back-EMF observer with a 16-bit fractional input D-Q current and voltage, and a 16-bit electrical speed. All are within the range &lt;-1 ; 1).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.3.2 AMCLIB_BEMF_OBSRV_DQ_T_A32 type description

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Data type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>sEObsrv</td>
<td>GMCLIB_2COOR_DQ_T_F32</td>
<td>Estimated back-EMF voltage structure.</td>
</tr>
<tr>
<td>sIObsrv</td>
<td>GMCLIB_2COOR_DQ_T_F32</td>
<td>Estimated current structure.</td>
</tr>
<tr>
<td>sCtrl</td>
<td>frac32_t</td>
<td>State variable in the alpha part of the observer, integral part at step k - 1. The variable is within the range &lt;-1 ; 1).</td>
</tr>
<tr>
<td>f32IQ_1</td>
<td>frac32_t</td>
<td>State variable in the beta part of the observer, integral part at step k - 1. The variable is within the range &lt;-1 ; 1).</td>
</tr>
<tr>
<td>a32PGain</td>
<td>acc32_t</td>
<td>The observer proportional gain is set up according to Equation 22 on page 31 as: $2\zeta\omega_0 L_D - R_S$ The parameter is within the range &lt;0 ; 65536.0). Set by the user.</td>
</tr>
<tr>
<td>a32IGain</td>
<td>acc32_t</td>
<td>The observer integral gain is set up according to Equation 22 on page 31 as: $\omega_0^2 L_D$ The parameter is within the range &lt;0 ; 65536.0). Set by the user.</td>
</tr>
<tr>
<td>a32IGain</td>
<td>acc32_t</td>
<td>The current coefficient gain is set up according to Equation 20 on page 30 as: $\frac{L_D}{L_D + T_s R_S}$ The parameter is within the range &lt;0 ; 65536.0). Set by the user.</td>
</tr>
<tr>
<td>a32UGain</td>
<td>acc32_t</td>
<td>The voltage coefficient gain is set up according to Equation 20 on page 30 as: $\frac{T_s}{L_D + T_s R_S} \frac{u_{max}}{i_{max}}$</td>
</tr>
</tbody>
</table>

*Table continues on the next page...*
### 2.3.3 Declaration

The available AMCLIB_PMSMBemfObsrvDQInit functions have the following declarations:

```c
void AMCLIB_PMSMBemfObsrvDQInit_F16(AMCLIB_BEMF_OBSRV_DQ_T_A32 *psCtrl)
```

The available AMCLIB_PMSMBemfObsrvDQ functions have the following declarations:

```c
frac16_t AMCLIB_PMSMBemfObsrvDQ_F16(const GMCLIB_2COOR_DQ_T_F16 *psIDQ, const GMCLIB_2COOR_DQ_T_F16 *psUDQ, frac16_t f16Speed, AMCLIB_BEMF_OBSRV_DQ_T_A32 *psCtrl)
```

### 2.3.4 Function use

The use of the AMCLIB_PMSMBemfObsrvDQ function is shown in the following example:

```c
#include "amclib.h"

static GMCLIB_2COOR_DQ_T_F16 sIdq, sUdq;
static AMCLIB_BEMF_OBSRV_DQ_T_A32 sBemfObsrv;
static frac16_t f16Speed, f16Error;

void Isr(void);

void main (void)
```
Initialization of the observer's structure
AMCLIB_PMSMBemfObsrvDQInit_F16(&sBemfObsrv);

BEMF Observer calculation
f16Error = AMCLIB_PMSMBemfObsrvDQ_F16(&sIdq, &sUdq, f16Speed, &sBemfObsrv);
Appendix A
Library types

A.1 bool_t

The bool_t type is a logical 16-bit type. It is able to store the boolean variables with two states: TRUE (1) or FALSE (0). Its definition is as follows:

```
typedef unsigned short bool_t;
```

The following figure shows the way in which the data is stored by this type:

<table>
<thead>
<tr>
<th>Value</th>
<th>15</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUE</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>FALSE</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To store a logical value as bool_t, use the FALSE or TRUE macros.

A.2 uint8_t

The uint8_t type is an unsigned 8-bit integer type. It is able to store the variables within the range <0 ; 255>. Its definition is as follows:

```
typedef unsigned char int8_t;
```

The following figure shows the way in which the data is stored by this type:
A.3  

Table A-2. Data storage

<table>
<thead>
<tr>
<th>Value</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>255</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
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<tr>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>F</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textbf{A.3  uint16\_t}

The \texttt{uint16\_t} type is an unsigned 16-bit integer type. It is able to store the variables within the range \(<0 \; ; \; 65535\>). Its definition is as follows:

\begin{verbatim}
typedef unsigned short uint16_t;
\end{verbatim}

The following figure shows the way in which the data is stored by this type:

Table A-3. Data storage

<table>
<thead>
<tr>
<th>Value</th>
<th>15</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>65535</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
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<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>15518</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
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<td>0</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>40768</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>F</td>
<td>4</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A.4  

Table A-4. Data storage

<table>
<thead>
<tr>
<th>Value</th>
<th>15</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>65535</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>15518</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40768</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>F</td>
<td>4</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The `uint32_t` type is an unsigned 32-bit integer type. It is able to store the variables within the range `<0 ; 4294967295>`. Its definition is as follows:

```c
typedef unsigned long uint32_t;
```

The following figure shows the way in which the data is stored by this type:

```
+----------------+-----------------+-----------------+-----------------+-----------------+-----------------+
| Value           | 31   | 24  | 23  | 16  | 15  | 8   | 7   | 0   |
|----------------+------|------|------|------|------|------|------|------|
| 4294967295     | F    | F   | F   | F   | F   | F   | F   | F   |
| 2147483648     | 8    | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 55977296       | 0    | 3   | 5   | 6   | 2   | 5   | 5   | 0   |
| 3451051828     | C    | D   | B   | 2   | D   | F   | 3   | 4   |
+----------------+------|------|------|------|------|------|------|------|
```

### A.5 `int8_t`

The `int8_t` type is a signed 8-bit integer type. It is able to store the variables within the range `<-128 ; 127>`. Its definition is as follows:

```c
typedef char int8_t;
```

The following figure shows the way in which the data is stored by this type:

```
+----------------+-----------------+-----------------+-----------------+-----------------+-----------------+
| Value           | 7    | 6   | 5   | 4   | 3   | 2   | 1   | 0   |
|----------------+------|------|------|------|------|------|------|------|
| 127             | 0    | 1   | 1   | 1   | 1   | 1   | 1   | 1   |
|                | 7    | F   |     |     |     |     |     |     |
| -128            | 1    | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|                | 8    | 0   |     |     |     |     |     |     |
| 60              | 0    | 0   | 1   | 1   | 1   | 1   | 0   | 0   |
|                | 3    | C   |     |     |     |     |     |     |
| -97             | 1    | 0   | 0   | 1   | 1   | 1   | 1   | 1   |
|                | 9    | F   |     |     |     |     |     |     |
+----------------+------|------|------|------|------|------|------|------|
```
A.6 int16_t

The int16_t type is a signed 16-bit integer type. It is able to store the variables within the range <-32768 ; 32767>. Its definition is as follows:

typedef short int16_t;

The following figure shows the way in which the data is stored by this type:

<table>
<thead>
<tr>
<th>Table A-6. Data storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>32767</td>
</tr>
<tr>
<td>-32768</td>
</tr>
<tr>
<td>15518</td>
</tr>
<tr>
<td>-24768</td>
</tr>
</tbody>
</table>

A.7 int32_t

The int32_t type is a signed 32-bit integer type. It is able to store the variables within the range <-2147483648 ; 2147483647>. Its definition is as follows:

typedef long int32_t;

The following figure shows the way in which the data is stored by this type:

<table>
<thead>
<tr>
<th>Table A-7. Data storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>2147483647</td>
</tr>
<tr>
<td>-2147483648</td>
</tr>
<tr>
<td>55977296</td>
</tr>
<tr>
<td>-843915468</td>
</tr>
</tbody>
</table>
A.8 frac8_t

The frac8_t type is a signed 8-bit fractional type. It is able to store the variables within the range <-1 ; 1). Its definition is as follows:

typedef char frac8_t;

The following figure shows the way in which the data is stored by this type:

<table>
<thead>
<tr>
<th>Value</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99219</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-1.0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.46875</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.75781</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

To store a real number as frac8_t, use the FRAC8 macro.

A.9 frac16_t

The frac16_t type is a signed 16-bit fractional type. It is able to store the variables within the range <-1 ; 1). Its definition is as follows:

typedef short frac16_t;

The following figure shows the way in which the data is stored by this type:

<table>
<thead>
<tr>
<th>Value</th>
<th>15</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99997</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-1.0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table continues on the next page...
To store a real number as \texttt{frac16\_t}, use the \texttt{FRAC16} macro.

### A.10 \texttt{frac32\_t}

The \texttt{frac32\_t} type is a signed 32-bit fractional type. It is able to store the variables within the range \(-1 ; 1\). Its definition is as follows:

```c
typedef long frac32_t;
```

The following figure shows the way in which the data is stored by this type:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Value & 31 & 24 & 23 & 16 & 15 & 8 8 7 0 \\
\hline
0.9999999995 & 7 & F & F & F & F & F \\
-1.0 & 8 & 0 & 0 & 0 & 0 & 0 \\
0.02606645970 & 0 & 3 & 5 & 6 & 2 & 5 \\
-0.3929787632 & C & D & B & 2 & D & F \\
\hline
\end{tabular}
\caption{Data storage}
\end{table}

To store a real number as \texttt{frac32\_t}, use the \texttt{FRAC32} macro.

### A.11 \texttt{acc16\_t}

The \texttt{acc16\_t} type is a signed 16-bit fractional type. It is able to store the variables within the range \((-256 ; 256\). Its definition is as follows:

```c
typedef short acc16_t;
```

The following figure shows the way in which the data is stored by this type:
### Table A-11. Data storage

<table>
<thead>
<tr>
<th>Value</th>
<th>Sign</th>
<th>Integer</th>
<th>Fractional</th>
</tr>
</thead>
<tbody>
<tr>
<td>255.9921875</td>
<td>0</td>
<td>1 1 1 1</td>
<td>F F F</td>
</tr>
<tr>
<td>-256.0</td>
<td>1</td>
<td>0 0 0 0</td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>0 0 0 0</td>
<td>1 0 0 0</td>
</tr>
<tr>
<td>-1.0</td>
<td>1</td>
<td>1 1 1 1</td>
<td>1 1 0 0</td>
</tr>
<tr>
<td>13.7890625</td>
<td>F</td>
<td>F F F F</td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>-89.71875</td>
<td>D</td>
<td>3 2 4 5</td>
<td>B C</td>
</tr>
</tbody>
</table>

To store a real number as acc16_t, use the ACC16 macro.

### A.12 acc32_t

The acc32_t type is a signed 32-bit accumulator type. It is able to store the variables within the range <-65536 ; 65536). Its definition is as follows:

```c
typedef long acc32_t;
```

The following figure shows the way in which the data is stored by this type:

### Table A-12. Data storage

<table>
<thead>
<tr>
<th>Value</th>
<th>S</th>
<th>Integer</th>
<th>Fractional</th>
</tr>
</thead>
<tbody>
<tr>
<td>65535.999969</td>
<td>7</td>
<td>F F F F</td>
<td>F F F F</td>
</tr>
<tr>
<td>-65536.0</td>
<td>8</td>
<td>0 0 0 0</td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>0 0 0 0</td>
<td>8 0 0 0</td>
</tr>
<tr>
<td>-1.0</td>
<td>F</td>
<td>F F F F</td>
<td>8 0 0 0</td>
</tr>
<tr>
<td>23.789734</td>
<td>0</td>
<td>0 0 0 B</td>
<td>E 5 1 6</td>
</tr>
<tr>
<td>-1171.306793</td>
<td>F</td>
<td>D B 6</td>
<td>5 8 B C</td>
</tr>
</tbody>
</table>

To store a real number as acc32_t, use the ACC32 macro.
A.13 GMCLIB_3COOR_T_F16

The GMCLIB_3COOR_T_F16 structure type corresponds to the three-phase stationary coordinate system, based on the A, B, and C components. Each member is of the frac16_t data type. The structure definition is as follows:

```c
typedef struct
{
    frac16_t f16A;
    frac16_t f16B;
    frac16_t f16C;
} GMCLIB_3COOR_T_F16;
```

The structure description is as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>frac16_t</td>
<td>f16A</td>
<td>A component; 16-bit fractional type</td>
</tr>
<tr>
<td>frac16_t</td>
<td>f16B</td>
<td>B component; 16-bit fractional type</td>
</tr>
<tr>
<td>frac16_t</td>
<td>f16C</td>
<td>C component; 16-bit fractional type</td>
</tr>
</tbody>
</table>

A.14 GMCLIB_2COOR_ALBE_T_F16

The GMCLIB_2COOR_ALBE_T_F16 structure type corresponds to the two-phase stationary coordinate system, based on the Alpha and Beta orthogonal components. Each member is of the frac16_t data type. The structure definition is as follows:

```c
typedef struct
{
    frac16_t f16Alpha;
    frac16_t f16Beta;
} GMCLIB_2COOR_ALBE_T_F16;
```

The structure description is as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>frac16_t</td>
<td>f16Alpha</td>
<td>α-component; 16-bit fractional type</td>
</tr>
<tr>
<td>frac16_t</td>
<td>f16Beta</td>
<td>β-component; 16-bit fractional type</td>
</tr>
</tbody>
</table>
A.15 GMCLIB_2COOR_DQ_T_F16

The GMCLIB_2COOR_DQ_T_F16 structure type corresponds to the two-phase rotating coordinate system, based on the D and Q orthogonal components. Each member is of the frac16_t data type. The structure definition is as follows:

```c
typedef struct
{
    frac16_t f16D;
    frac16_t f16Q;
} GMCLIB_2COOR_DQ_T_F16;
```

The structure description is as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>frac16_t</td>
<td>f16D</td>
<td>D-component; 16-bit fractional type</td>
</tr>
<tr>
<td>frac16_t</td>
<td>f16Q</td>
<td>Q-component; 16-bit fractional type</td>
</tr>
</tbody>
</table>

A.16 GMCLIB_2COOR_DQ_T_F32

The GMCLIB_2COOR_DQ_T_F32 structure type corresponds to the two-phase rotating coordinate system, based on the D and Q orthogonal components. Each member is of the frac32_t data type. The structure definition is as follows:

```c
typedef struct
{
    frac32_t f32D;
    frac32_t f32Q;
} GMCLIB_2COOR_DQ_T_F32;
```

The structure description is as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>frac32_t</td>
<td>f32D</td>
<td>D-component; 32-bit fractional type</td>
</tr>
<tr>
<td>frac32_t</td>
<td>f32Q</td>
<td>Q-component; 32-bit fractional type</td>
</tr>
</tbody>
</table>

A.17 GMCLIB_2COOR_SINCOS_T_F16
The `GMCLIB_2COOR_SINCOS_T_F16` structure type corresponds to the two-phase coordinate system, based on the Sin and Cos components of a certain angle. Each member is of the `frac16_t` data type. The structure definition is as follows:

```c
typedef struct
{
    frac16_t f16Sin;
    frac16_t f16Cos;
} GMCLIB_2COOR_SINCOS_T_F16;
```

The structure description is as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>frac16_t</td>
<td>f16Sin</td>
<td>Sin component; 16-bit fractional type</td>
</tr>
<tr>
<td>frac16_t</td>
<td>f16Cos</td>
<td>Cos component; 16-bit fractional type</td>
</tr>
</tbody>
</table>

### A.18 FALSE

The `FALSE` macro serves to write a correct value standing for the logical FALSE value of the `bool_t` type. Its definition is as follows:

```c
#define FALSE    ((bool_t)0)
```

```c
#include "mlib.h"

static bool_t bVal;

void main(void)
{
    bVal = FALSE;               /* bVal = FALSE */
}
```

### A.19 TRUE

The `TRUE` macro serves to write a correct value standing for the logical TRUE value of the `bool_t` type. Its definition is as follows:

```c
#define TRUE     ((bool_t)1)
```

```c
#include "mlib.h"

static bool_t bVal;
```
void main(void)  
{  
    bVal = TRUE;               /* bVal = TRUE */
}  

A.20  FRAC8

The FRAC8 macro serves to convert a real number to the frac8_t type. Its definition is as follows:

#define FRAC8(x) ((frac8_t)((x) < 0.9921875 ? ((x) >= -1 ? (x)*0x80 : 0x80) : 0x7F))

The input is multiplied by 128 (=2^7). The output is limited to the range <0x80 ; 0x7F>, which corresponds to <-1.0 ; 1.0-2^-7>.

#include "mlib.h"
static frac8_t f8Val;
void main(void)  
{  
    f8Val = FRAC8(0.187);               /* f8Val = 0.187 */
}  

A.21  FRAC16

The FRAC16 macro serves to convert a real number to the frac16_t type. Its definition is as follows:

#define FRAC16(x) ((frac16_t)((x) < 0.999969482421875 ? ((x) >= -1 ? (x)*0x8000 : 0x8000) : 0x7FFF))

The input is multiplied by 32768 (=2^15). The output is limited to the range <0x8000 ; 0x7FFF>, which corresponds to <-1.0 ; 1.0-2^-15>.

#include "mlib.h"
static frac16_t f16Val;
void main(void)  
{  
    f16Val = FRAC16(0.736);               /* f16Val = 0.736 */
}  

A.22 FRAC32

The FRAC32 macro serves to convert a real number to the frac32_t type. Its definition is as follows:

```
#define FRAC32(x) ((frac32_t)((x) < 1 ? ((x) >= -1 ? (x)*0x80000000 : 0x80000000) : 0x7FFFFFFF))
```

The input is multiplied by 2147483648 (=2^{31}). The output is limited to the range <0x80000000 ; 0x7FFFFFFF>, which corresponds to <-1.0 ; 1.0-2^{-31}>.

```c
#include "mlib.h"
static frac32_t f32Val;
void main(void)
{
    f32Val = FRAC32(-0.1735667);               /* f32Val = -0.1735667 */
}
```

A.23 ACC16

The ACC16 macro serves to convert a real number to the acc16_t type. Its definition is as follows:

```
#define ACC16(x) ((acc16_t)((x) < 255.9921875 ? ((x) >= -256 ? (x)*0x80 : 0x8000) : 0x7FFF))
```

The input is multiplied by 128 (=2^7). The output is limited to the range <0x8000 ; 0x7FFF> that corresponds to <-256.0 ; 255.9921875>.

```c
#include "mlib.h"
static acc16_t a16Val;
void main(void)
{
    a16Val = ACC16(19.45627);               /* a16Val = 19.45627 */
}
```

A.24 ACC32

The ACC32 macro serves to convert a real number to the acc32_t type. Its definition is as follows:
#define ACC32(x) ((acc32_t)((x) < 65535.999969482421875 ? ((x) >= -65536 ? (x)*0x8000 : 0x80000000) : 0x7FFFFFFF))

The input is multiplied by 32768 (=2^{15}). The output is limited to the range <0x80000000 ; 0x7FFFFFFF>, which corresponds to <-65536.0 ; 65536.0-2^{-15}>.

#include "mlib.h"

static acc32_t a32Val;

void main(void)
{
    a32Val = ACC32(-13.654437);               /* a32Val = -13.654437 */
}